ADVANCED OPTION PRICING MODELS

An Empirical Approach to Valuing Options

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This book is dedicated to those who apply the knowledge and use their success to do some good in the world.

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INTRODUCTION

This book is the result of years of original scientific research into the various elements that are required to accurately price options. We approached the topic in an objective and systematic manner, just as we did in our study of futures trading systems in *The Encyclopedia of Trading Strategies* (Katz and McCormick, 2000). The method: a traditional labor- and data-intensive study involving thousands of hours of computer time; the result: a wealth of practical findings of direct relevance to those who use options to speculate or hedge.

An in-depth investigation was necessary because of the nature of the subject under study. As is well known, options are a fiercely competitive, zero-sum game. The amateur usually does not stand a chance and even experienced players can find it difficult to use them effectively. Therefore, to successfully speculate or hedge with options, every edge is necessary. As in almost all realms of endeavor, knowledge can provide the biggest edge. A thorough, clear-sighted understanding of the subject and the factors that influence it are critical and can only be achieved by implementing an objective, scientific approach.

When it comes to options, knowledge can mean the difference between making a profit and taking a loss. For example, there is great advantage in knowing how to identify and exploit mispriced options. We are not referring to small mispricings that only the most efficient arbitrageur or market maker can exploit, but to gross mispricings that sometimes appear and, equally quickly, disappear. There is an edge in knowing what to look for and in knowing how to find it.

We know we must search for gross mispricings, but how do we find them? An option pricing model is needed. However, not just any pricing model will do. To gain an edge, a model must correctly value options under circumstances that cause standard models to break down. In addition, the model must be used with valid inputs; even the best model will not yield accurate prices if the model's inputs are in error.

In this book, we have done the research, described the logic behind it and the steps involved, and presented the results as practical solutions. We have analyzed standard option pricing models, discovered their flaws, and investigated better estimators of volatility and other model inputs. We have also explored nonstandard, rather innovative ways to achieve more accurate appraisals of option value. It is our sincere hope that this will give you the edge you need in the tough options game.

THINKING OUT OF THE BOX

Basically, there are four kinds of books on the subject of options: (1) those that deal with the basics and the strategic use of options, (2) academic texts that discuss theoretical models of how stock returns are generated, models that are then used to construct option pricing formulae, (3) practical guides that provide advice (derived largely from personal experience) on how to grab profits from the markets, and then there is (4) the book you are holding in your hands.

Options as a Strategic Investment by McMillan (1993) represents a well-written, classic example of the first kind of book. Such texts provide a basic understanding of options: they cover Black-Scholes and the Greeks, discuss the effects of time decay and of price changes in the underlying stocks, and contain everything you would ever want to know about covered writes, naked puts, straddles, spreads, equivalent positions, arbitrage, and the risk profiles of various positions. Such background information can help one maneuver intelligently around the options markets, as well as tailor positions to fit individual needs and expectations. However, unless you are one of the rare few who can divine the future behavior of the markets, relying only on texts such as these will not provide the practical knowledge you need to gain a statistical edge.

The second kind of book, the academically-oriented text, is heavy in theory. Such literature mostly consists of the kind of material that has been implemented in the computer programs used daily by market makers and options traders, or of esoterica that is primarily of interest only to academic theoreticians. An example of a good book of this genre is *Black-Scholes and Beyond* by Chriss (1997). Most of these texts present detailed theoretical analyses of option pricing models like Black-Scholes and Cox-Ross-Rubinstein, as well as variations thereon. If you are like Katz, an individual who enjoys playing with theoretical models and running Monte Carlo simulations, then you will find such books a lot of fun. Active options traders concerned with their bottom lines, however, probably will not greatly benefit from such reading.

Good books of the third kind are rather rare. Perhaps the best example of this kind of publication is *McMillan on Options* (McMillan, 1996). Such books are often based on personal experience and get down to the nitty-gritty by demonstrating how to take profits from the options markets. They cover topics like how volume and implied volatility can be tip-offs to large and profitable moves, how to interpret and profit from extremes in volatility, how to recognize significant situations, and much more. In terms of providing hard-hitting, practical insight for the active options trader, such books can be extremely useful.

The last category of book is, as far as we know, occupied solely by ours. This distinction is the result of taking a unique perspective on the subject. Although, as in academic texts, we discuss distributions of returns, the Central Limit Theorem, and random walks, in our book there is a heavier-than-usual emphasis on the empirical. Many books on option pricing focus on theoretical models and only use data in an effort to test the assumptions made by these models. Rather than discuss *theoretical* distributions of price changes, we examine *actual, real-market* distributions and how they differ from the theoretical distributions assumed by such popular pricing models as Black-Scholes. By using this approach we provide far more extensive coverage of real-market behavior than do most texts on pricing models, together with a wealth of data and analyses not available elsewhere. In our examination of distributions, we search for and find practical information about pricing that anyone can use to take money out of the markets. In short, we provide a detailed exploration of standard assumptions and then demonstrate how and where they are violated by real-market behavior. Our integrative approach leads to insights, not merely of theoretical significance, but of practical value for the trader trying to pull money from the options markets. The information presented is otherwise hard to come by, but essential to anyone wishing to become successful in the highly competitive arena of options trading.

IMPROVING OPTION PRICING STRATEGIES: A SCIENTIFIC INVESTIGATION

In this book, we are exploring a lot of new ground. The emphasis is on asking a wide range of questions and attempting to find the answers by studying real-market stock and options data. The ultimate goal is to find new and effective techniques for modeling the price movements of stocks and the value of the options that trade on them. At the same time, we investigate all the factors that bear upon the pricing of options, examine the standard pricing models, and discover where those models go wrong and lead to mispricing. The outcome: more effective ways to price options. We not only look at the subject from a theoretical point of view, but also study the actual movements of prices in the market, how they are distributed, and what the patterns tell us.

Our approach is empirical and analytic; our style is intuitive and practical. The work is based on continuing investigations by the authors who are themselves options traders. Much of this work involves extensive examination of long-persistent market characteristics and in-depth statistical and mathematical analyses. We present solid, research-based information of a kind often buried in academic journals, but do so in a manner that is immediately and practically useful. The analysis and results should be as valid and relevant many years from now as they are today.

ASSUMPTIONS MADE BY POPULAR MODELS: ARE THEY CORRECT?

In the world of equity options, accurate, mathematically-based estimates of fair price and of the so-called "Greeks" are crucial for success. Many professionals, as well as amateurs, still use the traditional Black-Scholes model to price options. Likewise, most standard texts on the subject focus primarily on Black-Scholes, while occasionally discussing Cox-Ross-Rubinstein and other related models. A common feature of these models is the assumption that, on a logarithmic scale, the distribution of returns (profits or losses) in the market is normal (Black-Scholes), something close to normal, or something that approaches normal in the limit (Cox-Ross-Rubinstein). Most "random walkers"—proponents of the Efficient Market Hypothesis (EMH)—would argue that the assumption of normally distributed returns is justified by the Central Limit Theorem and the "fact" that stock returns reflect the accumulation of large numbers of equally small, random movements. However, do stock returns really follow the familiar bell-shaped curve of the normal distribution? No, they do not!

It is well known (and easy to verify) that empirical distributions of returns deviate from normal by, at the very least, having longer tails—extreme returns are more frequent than would be expected from a normal or near-normal distribution. This does not necessarily imply that price movements are following something other than a random walk. Perhaps the "small, random movements" of the walk are simply not homogeneous. If some random steps are drawn from a different underlying distribution than others, e.g., a distribution that has a larger variance, a long-tailed distribution of returns might result. Of course, the EMH itself might be in error; perhaps stock prices are not random, but have memory and move in trends. Again, the result could be a long-tailed distribution of returns. Regardless of the reason, the difference between the empirical and normal distributions has significant implications for option pricing. Frankly, models that assume normality (like Black-Scholes) cannot be trusted to consistently provide correct option prices and, therefore, can cost hedgers and traders serious money!

Some might argue that, although the assumptions underlying Black-Scholes and related models are technically violated, the prices generated by these models approximate correct values well enough for practical purposes. Indeed, in many instances, they do. However, it is easy to find conditions under which the prices, and other data that are generated by a model like Black-Scholes, dramatically miss the mark. Consider the so-called "volatility smile" that has been the subject of many academic papers. The smile appears when implied volatility is plotted against strike price: deeply in- or out-of-the-money options have higher implied volatilities than at-the-money options. If we take a somewhat different perspective, when using Black-Scholes deeply in- or out-of-the-money options appear overpriced relative to at-the-money options. This is exactly what would be expected when a model that assumes a normal, short-tailed distribution of returns is applied to markets with long-tailed distributions of returns. In addition, due to the mean-reverting nature of volatility, pricing errors become substantial when historical volatility is an input to the model and reaches either very high or very low levels. A number of other statistical features of the underlying security can also result in seriously mispriced options.

The errors mentioned above are not small and of interest only to academicians. Under certain circumstances, many of these errors reach a magnitude that is quite significant, even to the average options trader. Because of the popularity and naive use of Black-Scholes and similar models, options can often be found trading near the model's estimate of fair value when, in fact, they should be priced substantially higher or lower. A savvy options player can take advantage of the discrepancies and sell the overpriced options or buy the underpriced ones. A more sophisticated and realistic pricing model—one that takes into account the actual distributions of returns seen in the market under various conditions and that is less subject to systematic error—can be a powerful weapon for the trader or hedger seeking a decisive edge over his or her competitors. Can such a model be developed? What is involved in the construction of an improved pricing model? Finally, assuming it can be built, how will such a model perform in the competitive world of equity options?

This book examines the specific conditions under which Black-Scholes and other popular models fail to provide good estimates of fair option prices, the actual behavior of the market and how it differs from what the standard models assume, how to use such information to arrive at better option price estimates, and the steps involved in building better option pricing models.

OPTIMAL MODEL INPUTS

In addition to the pricing model itself, certain inputs require special attention. As anyone familiar with options knows, volatility is one of the major factors that determine the value of an option.

When using an option pricing model, historical volatility is often employed as one of the inputs. In such a case, historical volatility is being used (knowingly or not) as a proxy for future volatility. It is actually future volatility, not historical volatility, that determines the worth of an option. Therefore, the direct use of historical volatility as an input to a standard model can lead to systematic and often severe mispricing. To some extent, volatility appears to be mean-reverting. If recent historical volatility is extremely high, one can expect future volatility to be lower; if recent historical volatility is extremely low, future volatility can be expected to be higher. As it is future volatility that matters, the use of historical volatility can distort option price appraisals: extremely high historical volatility can lead to overpricing and extremely low historical volatility can cause the model to underprice options. The use of estimated future volatility, rather than simple historical volatility, can improve price estimates based on any option pricing model. Fortunately, volatility is much more predictable than price movement and, as we will show, predictive models can be constructed for it.

A substantial amount of space has been dedicated to the prediction and estimation of volatility, considering its great importance to the pricing of options. Implied volatility is also examined in detail. Finally, the way in which time (another major determinant of an option's value) and volatility are related, both in theory and in actual practice, is studied.

WHAT IS COVERED IN THE CHAPTERS?

The book begins with coverage of the basics of options, fair value, and pricing models. Chapter 1 provides a brief, but detailed, review of options and option terminology. A clear discussion of the Greeks and the use of standard option pricing models is included. In addition, the basics of speculation, one form of arbitrage, and equivalent positions are reviewed. The use of option characteristic curves, or price response charts, is also illustrated.

Chapter 2 attempts to elucidate the nature of fair value. What is fair value? How is fair value related to the efficient market hypothesis? Is fair value a unitary entity or a multiheaded beast and, if it is the latter, what are its heads or components? The chapter considers fair value in terms of both speculation on future prices and certain kinds of arbitrage. A simple Monte Carlo experiment, in which synthetic stock and option prices are generated and examined, is presented to illustrate some of the concepts developed in this chapter. The illustrative model examined in the experiment is the starting point that, with modifications, becomes a real option pricing model in the next chapter.

Chapter 3 contains an examination of the two most popular option pricing models: Black-Scholes and Cox-Ross-Rubinstein. These two models have such a pervasive presence in the world of options that their influence on prices, and the behavior of traders and hedgers, is overwhelming. The assumptions on which these models are based are investigated and the models themselves developed, illustrated, and dissected. A common thread in the assumptions underlying these models is discovered and analyzed. In this chapter, there is an extensive discussion of the log-normal distribution and its impact when used as a basis for understanding the underlying stock price movements or returns, as it indeed is used in the standard pricing models. Cox-Ross-Rubinstein (also known as the "binomial model") is fully developed and illustrated with both the Monte Carlo method and with pricing trees. The Black-Scholes equations are also presented and some interesting features of these equations (such as the fact that they are direct calculations of the expectation of future option prices, under the assumption of a log-normal distribution of returns) are demonstrated numerically with the aid of numerical quadrature. Finally, some phenomena associated with log-normally distributed stock price movements are discussed; specifically, the fact that if there is an even probability of either a win or a loss, there must be a positive net return, and if there is an average return that is neither positive nor negative (i.e., a return that is zero or breakeven), then the probability of any stock trade taking a loss must be greater than 50%, all this being true if stock prices are indeed log-normal random walks. The chapter concludes with an examination of stock price movements in the NASD and NYSE, as well as those generated in the course of a Monte Carlo experiment and designed to behave according to the log-normal random walk assumption.

After the heavily theoretical discussion in Chapter 3, the orientation becomes empirical.

Chapter 4 studies the distribution of actual stock returns by examining their statistical moments. The reason for studying stock price returns from the perspective of moments is to better characterize the distributions involved. Distributions of underlying stock price movement are a major determinant of the worth of options trading on those stocks. The first four moments of a distribution are defined and discussed. Moments are useful statistics in characterizing the shape either of a theoretical distribution or of one constructed based on sample data. Once the basics are defined, the database used in all the studies that follow in this chapter is discussed, as are the basic software tools and methodology.

Chapter 4 also contains a series of empirical studies or tests in which a variety of questions are answered on the basis of an examination of the statistical moments of the distributions of returns. The study of moments can help determine, for example, whether the underlying distribution of price movements in stocks is indeed log-normal, as popular option models assume. If the distribution is not log-normal, moments can help characterize its shape and how it differs from the log-normal baseline. In this chapter, moments are examined in relationship to holding period, day of the week, time of the year, and time with respect to option expiration.

Although it may sound strange to study statistical moments, the reader who is familiar with options has already encountered the second moment, which is, in fact, volatility. Almost every trader is familiar with the first moment, which is simply the expected gain or loss over the holding period, i.e., the trend. The following are some of the questions asked and answered in Chapter 4: Does volatility scale with the square root of time? Are successive returns independent of one another or, equivalently, is the market efficient and unpredictable? Is the distribution of returns log-normal and, if not, how does it differ from log-normal? Does volatility vary with day of the week and time of the year? Most traders and hedgers would say that the answer to these questions is yes. If volatility does vary with time, which days are the most and least volatile; which times of the year are most and least volatile? And what about the other moments, like skew, kurtosis, and expectation? Finally, what does the characterization (in terms of moments) of the distribution of real-market returns reveal about the worth of options under a variety of different conditions?

Chapter 5 is dedicated to that statistical moment dear to the heart of every options player, whether speculator or hedger: volatility. When pricing options, the volatility of concern is not historical, but future; it is future volatility that can be expected to occur during the holding period. The focus of Chapter 5 is on the estimation or prediction of future volatility for the purpose of appraising options. This chapter is probably unlike any other chapter on volatility that you have read in any other book. The discussion begins with measurement reliability, as seen from the perspective of a psychometrician. Although psychometrics may seem far afield from the world of finance, it turns out that some of the problems involved are similar when abstracted from the specific content and require similar solutions. Some of the basics of psychometrics or "test theory" are discussed, such as estimating reliability using split-half correlations. Model complexity and other issues are then examined. At this point, the chapter covers the methodology employed, including the particular databases used, software involved, and the calculation of implied volatility (required in some of the studies). Then begins a series of tests concerned with various aspects of volatility.

Study 1 in Chapter 5 examines the common use of simple measures of historical volatility in pricing options. It asks a variety of questions. How good is historical volatility as a predictor of future volatility? Under what conditions does the use of historical volatility lead to serious pricing errors? Can historical volatility somehow be adjusted to yield better option appraisals? How reliable is historical volatility as a measure of the underlying trait of volatility possessed by a given stock at a given time?

Moreover, which of the many different measures of historical volatility should the trader or hedger employ? Although most users of standard models might not be aware of it, there are indeed a number of ways in which a measure of historical volatility may be obtained. The study leads to some interesting findings regarding the relationship between historical volatility and future market behavior and, in turn, the fair prices for options. One finding of critical importance is that the use of uncorrected or "raw" historical volatility can result in appraisals that are systematically distorted. In other words, standard models applied in the standard way, using historical volatility as one of the inputs, will, under certain conditions, yield theoretical fair prices that are far from the actual worth of the option being priced.

The goal in Study 2 is to determine whether the combined use of two different measures of historical volatility can improve the estimation of future volatility and, thus, of pricing accuracy. Here the technique of multivariate regression is employed. Some interesting charts are presented depicting the relationship between short- and long-term historical volatility and future volatility.

Study 3 is an in-depth analysis of the reliability of volatility measurements and the stability of the underlying volatility being measured. Here, the ingenious use of psychometric theory appears. Several kinds of volatility measures are considered and their reliability and validity assessed. Some surprising findings emerge—findings that can provide immediate benefit to the user of options.

Study 4 in Chapter 5 attempts to construct a more sophisticated estimator of future volatility; multivariate polynomial regression is employed. Inputs to the volatility forecasting model include historical volatility for two periods (using the most reliable measures found in the previous studies), as well as cycle harmonics to capture stable seasonal variations in volatility. The results are dramatically better estimates of future volatility. Regardless of the option pricing model used, this is the kind of volatility estimate that should be employed. This chapter does not include consideration of standard approaches to forecasting volatility, e.g., GARCH; such approaches and models have received extensive coverage by other authors. Instead, Chapter 5 embodies the spirit of this book, which is to think out of the box, to apply a variety of techniques that are not in general use, and to gain an edge, in terms of both simplicity and power.

Study 5 examines implied volatility as an estimator of future volatility. Again, some interesting findings emerge. Contrary to popular belief, implied volatility is not necessarily any better than historical volatility when used in a pricing model.

Finally, in Study 6, historical and implied volatility are together used to forecast future volatility. Again a technique, Sewall Wright's path analysis, is borrowed from another discipline that might seem to be far afield. Sewall Wright was a geneticist who explored correlations of traits that were passed on through generations. Path analysis allows causal inferences to be made from correlation matrices; these inferences concern the strength of a causal influence of one variable upon another when considered in the context of a number of variables and possible configurations of paths of causation. Path analysis helps answer questions like the following: to what extent is implied volatility determined (1) by historical volatility, and (2) by future volatility, perhaps as a result of the leakage of inside information?

Chapter 6 deals with pricing options using empirically-based conditional distributions. In standard models like Black-Scholes, theoretical distributions are assumed a priori. As has been demonstrated, the distributional assumptions made by such models often appear to be violated by the price behavior of real stocks; this leads to option pricing errors. What happens if the a priori distributions are replaced with distributions determined from real-market behavior? This is the central idea behind the use of conditional distributions. Various questions regarding the use of conditional distributions to price options are investigated.

One of the problems with conditional distributions derived from market data concerns curve-fitting and degrees of freedom. Chapter 6 begins with an extensive discussion of these issues, which includes the use of rescaling as a means of reducing the degrees of freedom consumed when constructing conditional distributions. General methodology, including data and software, are then briefly discussed. A series of empirical studies follow.

Study 1 explores a simple pricing model in which raw historical volatility is the only conditioning variable. Theoretical option premiums, determined from conditional distributions, are compared to Black-Scholes, with the latter model computed both with raw historical volatility and with an improved estimate based on raw historical volatility that corrected for nonlinearity and regression to the mean. Also presented are charts of theoretical premiums from Black-Scholes and from the empirical distribution methodology for several strikes.

Study 2 in Chapter 6 is essentially a replication of Study 1, except that raw historical volatility is replaced with a high quality estimate of future volatility.

In both Studies 1 and 2, the distributions employed are not detrended; any consistent trends, volatility-related or not, were allowed to influence theoretical option prices derived from the conditional distribution methodology. In Study 3, a reanalysis is performed with detrended distributions, i.e., the effect of trend is removed by adjusting the first moment of each distribution (its mean) to zero.

In Study 4, historical skew and kurtosis are added to the model as conditioning variables; they are computed in a manner similar to that used to compute historical volatility. The effect of skew and kurtosis on the worth of puts and calls at different levels of moneyness is examined.

Study 5 examines the effect of trading venue on the distributions of returns and, in turn, on option prices. Again, puts and calls, with varying strikes and moneyness, are examined and their empirically determined prices are compared to Black-Scholes.

In Study 6, distributions conditional upon the status of a popular technical indicator are computed and used to price options. Crossovers of the stochastic oscillator at the standard thresholds are examined. Although such indicators are of little use to speculative traders dependent on directional movement (see Katz and McCormick, 2000), they may be significant when trying to characterize aspects of the distribution of returns other than trend. The chapter concludes with a general discussion of the methodology, its strengths and weaknesses.

One of the problems with the use of conditional distributions is a heavy demand for massive amounts of data because the degrees of freedom required by the methodology can be enormous. One way of making the empirical approach to pricing options more workable is to employ a general nonlinear modeling technique that can smooth out the noise while still capturing the true relationships revealed by the conditional distribution methodology. In Chapter 7, nonlinear models that potentially have the ability to accomplish this are explored; specifically, neural networks, multivariate polynomial regressions, and hybrid models.

Chapter 7 begins with a detailed discussion of neural networks, multivariate polynomial regressions, and hybrid models. We cover everything from the issues of numerical stability and the accumulation of round-off errors to the use of Chebyshev Polynomials as a means of dealing with problems of colinearity.

A general problem with neural networks and multivariate polynomial regressions is the tendency to curve-fit the data. The use of a hybrid model is one possible solution to this problem. For example, a hybrid model might incorporate a neural network in which the output neuron behaves like Black-Scholes. The intention is to build into the model as much knowledge as possible, even if it is only approximate, and to do so in such a way that the errors in the approximation can be corrected for by various elements in the model. Black-Scholes, although exhibiting systematic error that can be great under certain conditions, does yield a reasonable first approximation to the worth of an option. What if some of the inputs to Black-Scholes could be tweaked to force it to yield more accurate appraisals? This is the idea behind the hybrid model under discussion. Why take the trouble of developing a hybrid model, rather than a simple neural network or polynomial regression? Because, with a hybrid model, the number of free parameters required to obtain a good fit to the data is substantially lower and, therefore, the solution much less prone to curve-fitting and the excessive consumption of degrees of freedom.

Before attempting to use nonlinear modeling techniques to price options based on real-market data, it is important to discover whether they could accurately emulate Black-Scholes. If a general nonlinear model cannot emulate Black-Scholes, how can it be expected to capture the possibly more complex relationship between factors such as volatility, time, and strike, and the fair premium of an option that might exist in real-market data? The first two studies of Chapter 7 answer this question.

In Study 1, a neural network is trained to emulate Black-Scholes and its performance is evaluated. In Study 2, a multivariate polynomial regression is fitted to the same Black-Scholes data set and evaluated for performance. Both approaches are demonstrated to be capable of doing a good job. Attention is then turned to real-market data.

Study 3 investigates the ability of a polynomial regression to accurately capture the relationships between fair premium and the model inputs that are seen in data derived from real stock returns using a methodological equivalent to conditional distributions. The behavior of the polynomial model is compared to Black-Scholes and is evaluated in terms of its ability to accurately describe the empirical pricing data, as well as to filter out random variations that are seen in such data. Several tables and charts illustrating the pricing behavior are presented.

Study 4 repeats Study 3, but uses a neural network instead of a multivariate polynomial.

Study 5 examines a hybrid model that consists of a neural network with a special Black-Scholes output neuron and some additional processing elements.

Chapter 7 concludes with a discussion that compares and evaluates the various approaches. The following kinds of questions are addressed. Which approach best captures the relationships required to accurately price options? Which approach is most susceptible to undesirable curve-fitting and which is most resistant? What are the specific problems that must be dealt with when attempting to develop models of this kind? And, what are the respective potentials of these various techniques when the goal is to develop a coherent and sophisticated option pricing model? Although long and elaborate, Chapter 7 is rather unique when it comes to the treatment of option pricing.

Chapter 8 revisits volatility. In this chapter, a wide range of variables beyond those explored in Chapter 5 are examined as potential predictors of future volatility. These variables include measures such as historical skew and kurtosis, and the status of various technical indicators. The aim is to obtain the best possible estimate of near-future volatility. Attempts are made to answer questions about volatility that were raised in the course of the other investigations in this book. For example, in Chapter 6, skew and kurtosis are found to affect fair premium. Do skew and kurtosis have an effect on premium that is mediated through volatility; in other words, do these variables influence the expectation regarding future volatility and, therefore, have some impact on the value of options? What about the status of technical indicators? Are their effects on option prices due to differences in future volatility, rather than just to differences in the shape of the distribution of future returns? The answers to these questions are found in Chapter 8.

So far, with few exceptions, theoretical option prices based on observed movements in stock prices have been the focus of our studies. In Chapter 9, comparisons are made between these theoretical option prices and real-market option prices, i.e., the prices at which the relevant options are actually trading. The chapter contains a discussion of the data and software used and attempts to answer a variety of questions concerning the relationship between option prices computed with different models to those observed in the actual marketplace. For example, when there is a wide disparity between the two figures, do real option prices fall closer to Black-Scholes or to what one of the better models suggests? Can one profit by looking for large discrepancies between the theoretical price of an option and the price at which it is actually trading? How much does the use of Black-Scholes and other popular models influence the options market?

Finally, there is the *Conclusion*. Here we summarize our findings and provide you with information on how these insights can improve your option pricing strategies.

WHO WILL BENEFIT?

This book is intended for everyone from the professional quant to the student who desires a better understanding of, and strategy for, pricing and trading options. Professional and institutional options players, who may be adjusting the standard models in an intuitive fashion, will find this book useful in that it may articulate their intuitive understanding of option pricing in such a way as to allow the automation or computerization of the pricing process. This could prove more effective than the intuitive approach, e.g., by leading to the inclusion of a wider range of conditions that will let hundreds, if not thousands, of options positions be quickly and repeatedly scanned for more frequent trading opportunities throughout the day. The sophisticated options player will also find this book helpful in that it will place him or her on a more level playing field with professional and institutional traders—we are giving you information about models that they may be using to obtain a closer estimate of an option's future value. The book should also be of interest to the academician or student trying to develop better theories and methods of option pricing. We expect that all readers will find within these pages at least one or two useful insights that will make their approach to option pricing more profitable.

Although this book contains a lot of mathematics and statistics, we make every effort to explain things—especially findings that are of practical use—in "plain English." We also include many tables and charts to illustrate the phenomena under discussion. Those who are mathematically challenged may want to ignore the equations and stick to the less technical text. Conversely, quants might want to skip the introductory material, like Chapter 1 on the basics.

TOOLS AND MATERIALS USED IN THE INVESTIGATION

As in any scientific investigation, the one essential element that is required is a subject of study. In the present case, that subject is the world of option pricing, as represented by real-market data. We used data from two sources: (1) stock price and volume data were obtained from the Worden Brothers TC-2000 database (www.worden.com); (2) option pricing, volume, and open interest data were obtained from www.stricknet.com.

In our investigation, we focus primarily on short-term equity options, with some attention to index options. The main reason for using short-term options in our study is because they are the most liquid and are the kind traded by the authors and most other active traders. Secondly, by working with short-term options, we can (somewhat safely) ignore interest rates and dividends, thereby simplifying our investigations. The simplifications, and their possible impact on the findings, will be discussed whenever they arise.

The tools used to investigate the subject come in the form of software. To a great extent, the software was custom written by Katz either exclusively for this book or for his company, Scientific Consultant Services, Inc. For example, *N-Train* (his neural network development system) was used in Chapter 7 as part of the study of nonlinear pricing models. Other custom developed software included libraries containing routines for Black-Scholes pricing, calculation of statistical moments, numerical quadrature, regression, data management, volatility calculations, general mathematics, pseudo-random number generators, probability functions, utility functions, and the *Neural-Hybrid Options Model Library*. Custom code was written for each of the tests and studies, as well as for actual pricing models, based on the resultant findings. At the end of this book, for the benefit of those who would like to replicate and expand on our work, we have provided information on how to obtain the *Companion CD* and other software used in our studies.

In addition, a number of off-the-shelf products were used in the course of the investigation. The GNU C/C++ and Fortran compilers familiar to Unix users were used to compile code. Microsoft's Excel spreadsheet was used for visualization (charting) and presentation, as well as for some final analyses. Lastly, routines from *Numerical Recipes in Fortran 77* and *Numerical Recipes in C* (both books and software packages by Press et al., 1992) were employed.

AN INVITATION

We invite you to visit our Web site: www.scientific-consultants.com. Here, you will find updates about our research and other information that you may find useful.

We also enjoy hearing from our readers; you are always welcome to send us your questions or comments by e-mail to katz@scientific-consultants.com and mccormick@scientific-consultants.com.

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CHAPTER 1

A Review of Options Basics

This chapter provides some background by defining options and many of the terms used when discussing them. It looks at where options are traded, examines a few of their salient characteristics, and discusses some of the factors that influence their value. The reader is introduced to option price charts, which are used to illustrate the option terminology and influential factors under discussion. The fundamentals of pricing models, the Greeks, put-call parity, synthetics, and equivalent positions are also covered. More advanced readers may wish to skip this chapter.

BASIC OPTIONS: CALLS AND PUTS

Options are contracts that come in two primary flavors: calls and puts. A *call* is an agreement that gives the holder (owner) the right to purchase the *underlying security* at a predetermined price called the *strike price* either at the call's *expiration date* (European-style options) or anytime during the life of the option (American-style options). If and when the holder of a call actually exploits that right, it is said that the call has been *exercised*. From the options seller's point of view, being short a call means having given someone the right to purchase from him or her the underlying security at the strike price of the option, regardless of that security's current market value. If and when this right is exercised, the trader who is short the option is said to have been *assigned*. While a call gives its holder the right to buy the underlying security, a *put* gives the trader the right to sell the underlying security at the strike price of the option. A trader who is short a put has an obligation to purchase the underlying security at the strike price of the option, should the put be exercised and he or she be assigned.

Options are *derivative securities* because they owe their existence and value to the underlying assets on which they trade. In this book, the underlying assets of the options examined are mostly individual stocks, although market and sector tracking entities such as the OEX index and the QQQ tracking stock are also studied. In other words, the focus here is primarily on *equity* (*stock) options* and *index options*. It should be noted that most stock options are American-style options, which may be exercised at any time prior to expiration. Index options may be either European style or American style.

Years ago, options contracts were traded "over the counter" and were customized to the contract writer's requirements. Today, while some options are still traded the old way, most have been standardized, are issued by the Option Clearing Corporation (OCC), and are traded on regulated exchanges as *listed* options. A standard, listed option is fully defined by the underlying security, expiration, strike price, and type (call or put). Standard stock and index options generally expire on the Saturday following the third Friday of the month of expiration. Consequently, when specifying the expiration, only the month and, for longer-term options such as LEAPS (long-term equity anticipation securities), the year need be stated. Standard stock options may be exercised until 5 p.m. Eastern Standard Time on the last day of trading. This can be a source of a nasty surprise, should the trader be caught off guard due to options being exercised after the close of the market. If assigned, a trader who is short will receive an assignment notice the day following the assignment. Finally, each standard stock options contract usually controls 100 shares of stock. An option on IBM that expires in May and gives the owner the right to buy 100 shares at $75 a share would be referred to as an "IBM May 75 call"; an option on IBM that expires in June and gives the holder the right to sell 100 shares at $52 a share would be specified as an "IBM June 52 put."

Options are traded on a number of exchanges. The Chicago Board Options Exchange (CBOE) was the first listed options exchange in the world. Their Web site at www.cboe.com contains a wealth of information on options that includes quotes, historical data, option analysis tools (including pricing models), and educational materials. A newer, electronic options trading venue is the International Securities Exchange (ISE). It offers excellent liquidity on many options, as well as fast executions. The authors trade the QQQ index options (often referred to as "Qubes") on the ISE. Options also trade on the American Stock Exchange (AMEX) and on several regional exchanges such as the Philadelphia Stock Exchange (PHLX) and the Pacific Stock Exchange (PSE).

NAKED AND COVERED

Options may be traded either *naked* or *covered*. A trader who sells a *covered call*, also known as a *covered write*, already owns the underlying security. For example, the owner of 100 shares of IBM might sell an IBM call, which entitles the buyer to "call away" the options seller's stock. A trader may also cover a short option position with a long position in another option on the same underlying security; the combined long-and-short position is known as a *spread*. Selling a *naked call* happens when a call is sold without owning the underlying stock. If the call happens to be exercised and the seller assigned, he or she will be obligated to sell the stock at the strike price of the option. This will result in a short position in the stock. The trader will probably want to quickly cover the short stock position and will have to do so by purchasing shares in the open market at a price higher than the strike price at which the short option position was established. A trader who sells a *naked put* is selling a put without being short the underlying security. If assigned, the seller is obligated to purchase the stock at the put's strike price, a price that is almost certain to be higher than the stock's current market value. Of course, if no assignment occurs, and the put or call expires worthless, the options seller gets to keep the entire premium he or she was paid for the option. Having options expire worthless is just what the seller of naked options usually hopes for, except when puts are sold as a means of acquiring stock at a lower cost basis.

ADDITIONAL OPTION TERMINOLOGY

The price of an option is its *premium*. Premium can be broken down into two components or kinds of value. One kind of value is *intrinsic value*. If you have an IBM call with a strike price of $100 and the stock is trading at $105, the option will have an intrinsic value of $5. This intrinsic value derives from the fact that if you exercise the option, you can buy the stock at $100 from the option's seller, then immediately turn around and sell the stock for $105 in the open market, pocketing a $5 profit. Options also have another kind of value that has to do with where the stock might go at some point in the future. Assume that the option has several months of life remaining before expiration. Its total worth is almost certain to be greater than $5; for instance, it may be trading at $7. In this example, the extra $2 is the so-called *time value* or *time premium* of the option. Time value derives from what might happen in the future. At some future point in the option's life, IBM stock might reach $200, in which case the profit that could be made from holding the call option would be at least $100. On the other hand, the stock could drop to $20, leaving the option holder with a nearly worthless option. But, if the option has any time remaining, it will still have some value since, at some point prior to expiration, the stock could again surge to over $100, the option's strike price.

An option is said to be *in-the-money* to the extent that it possesses intrinsic value. For a call to be in-the-money, the underlying security must be trading at a price that is greater than the strike price of the call. In such a case, the call's intrinsic value is equal to the price of the underlying asset minus the strike price of the option. Conversely, a put is in-the-money when the underlying security trades at a price lower than the option's strike price. An in-the-money put has an intrinsic value equal to the put's strike price minus the price of the underlying asset. An option is said to be *out-of-the-money* when it possesses no intrinsic value, only time value. A call is out-of-the-money when the underlying trades below the call's strike price, while