FACTS ON FILE SCIENCE LIBRARY

NOTABLE SCIENTISTS

A TO Z OF

Mathematicians

TUCKER McELROY, Ph.D.

A to Z of Mathematicians

NOTABLE SCIENTISTS

A TO Z OF MATHEMATICIANS

TUCKER MCELROY, Ph.D.

Facts On File, Inc.

A TO Z OF MATHEMATICIANS

Notable Scientists

Facts On File, Inc.
132 West 31st Street
New York NY 10001

Library of Congress Cataloging-in-Publication Data

McElroy, Tucker.
A to Z of mathematicians / Tucker McElroy.
p. cm.—(Notable scientists)
Includes bibliographical references and index.
ISBN 0-8160-5338-3 (acid-free paper)
1. Mathematicians—Biography. I. Title. II. Series.
QA28.M395 2005
510'.92'2—dc22
2004005460

Text design by Joan M. Toro
Cover design by Cathy Rincon
Chronology by Sholto Ainslie

Printed in the United States of America

VB TECHBOOKS 10 9 8 7 6 5 4 3 2 1

This book is printed on acid-free paper.

Contents

List of Entries

Acknowledgments

I thank God, who gives me strength and hope each day. I am grateful to my wife, Autumn, who encouraged me to complete this book. Thanks also go to Jim Dennison for title translations, as well as Diane Kit Moser and Lisa Yount for advice on photographs. Finally, my thanks go to Frank K. Darmstadt, Executive Editor, for his great patience and helpfulness, as well as the rest of the Facts On File staff for their work in making this book possible.

INTRODUCTION

A to Z of Mathematicians contains the fascinating biographies of 150 mathematicians: men and women from a variety of cultures, time periods, and socioeconomic backgrounds, all of whom have substantially influenced the history of mathematics. Some made numerous discoveries during a lifetime of creative work; others made a single contribution. The great Carl Gauss (1777–1855) developed the statistical method of least squares and discovered countless theorems in algebra, geometry, and analysis. Sir Isaac Newton (1643–1727), renowned as the primary inventor of calculus, was a profound researcher and one of the greatest scientists of all time. From the classical era there is Archimedes (287 B.C.E.–212 B.C.E.), who paved the way for calculus and made amazing investigations into mechanics and hydrodynamics. These three are considered by many mathematicians to be the princes of the field; most of the persons in this volume do not attain to the princes' glory, but nevertheless have had their share in the unfolding of history.

THE MATHEMATICIANS

A to Z of Mathematicians focuses on individuals whose historical importance is firmly established, including classical figures from the ancient Greek, Indian, and Chinese cultures as well as the plethora of 17th-, 18th-, and 19th-century mathematicians. I have chosen to exclude those born in the 20th century (with the exception of Kurt Gödel), so that the likes of Dame Mary Cartwright, Andrey Kolmogorov, and John Von Neumann are omitted; this choice reflects the opinion that true greatness is made lucid only through the passage of time. The earlier mathematicians were often scientists as well, also contributing to astronomy, philosophy, and physics, among other disciplines; however, the latter persons, especially those of the 19th century, were increasingly specialized in one particular aspect of pure or applied mathematics. Modern figures who were principally known for fields other than mathematics—such as Albert Einstein and Richard Feynman—have been omitted, despite their mathematical accomplishments. Being of the opinion that statistics is one of the mathematical sciences, I have included a smattering of great statisticians. Several sources were consulted in order to compile a diverse list of persons—a list that nevertheless delivers the main thrust of mathematical history.

I have attempted to make this material accessible to a general audience, and as a result the mathematical ideas are presented in simple terms that cut to the core of the matter. In some cases precision was sacrificed for accessibility. However, due to the abstruse nature of 19th- and 20th-century mathematics, many readers may still have difficulty. I suggest that they refer to Facts On File's handbooks in algebra, calculus, and geometry for unfamiliar terminology. It is helpful for readers to have knowledge of high school geometry and algebra, as well as calculus.

After each entry, a short list of additional references for further reading is provided. The majority of the individuals can be found in the *Dictionary of Scientific Biography* (New York, 1970–90), the *Encyclopaedia Britannica* (http://www.eb.com), and the online MacTutor History of Mathematics archive (http://www-gap.dcs.st-and.ac.uk/~history); so these references have not been repeated each time. In compiling references I tried to restrict sources to those articles written in English that were easily accessible to college undergraduates.

A

Abel, Niels Henrik

(1802–1829)
Norwegian
Algebra

The modest Norwegian mathematician Niels Abel made outstanding contributions to the theory of elliptic functions, one of the most popular mathematical subjects of the 19th century. Struggle, hardship, and uncertainty characterized his life; but under difficult conditions he still managed to produce a prolific and brilliant body of mathematical research. Sadly, he died young, without being able to attain the glory and recognition for which he had labored.

Niels Henrik Abel was born the son of Sören Abel, a Lutheran pastor, and Ane Marie Simonson, the daughter of a wealthy merchant. Pastor Abel's first parish was in the island of Finnöy, where Niels Abel was born in 1802. Shortly afterward, Abel's father became involved in politics.

Up to this time Abel and his brothers had received instruction from their father, but in 1815 they were sent to school in Oslo. Abel's performance at the school was marginal, but in 1817 the arrival of a new mathematics teacher, Bernt Holmboe, greatly changed Abel's fate. Holmboe recognized Abel's gift for mathematics, and they commenced studying LEONHARD EULER and the French mathematicians. Soon Abel had surpassed his teacher. At this time he was greatly interested in the theory of algebraic equations. Holmboe was delighted with his discovery of the young mathematician, and he enthusiastically acquainted the other faculty with the genius of Abel.

During his last year at school Abel attempted to solve the quintic equation, an outstanding problem from antiquity; but he failed (the equation has no rational solutions). Nevertheless, his efforts introduced him to the theory of elliptic functions. Meanwhile, Abel's father fell into public disgrace due to alcoholism, and after his death in 1820 the family was left in difficult financial circumstances.

Abel entered the University of Sweden in 1821, and was granted a free room due to his extreme poverty. The faculty even supported him out of its own resources; he was a frequent guest of the household of Christoffer Hansteen, the leading scientist at the university. Within the first year, Abel had completed his preliminary degree, allowing him the time to pursue his own advanced studies. He voraciously read everything he could find concerning mathematics, and published his first few papers in Hansteen's journal after 1823.

In summer 1823 Abel received assistance from the faculty to travel to Copenhagen, in

Niels Abel, one of the founders of the theory of elliptic functions, a generalization of trigonometric functions *(Courtesy of the Library of Congress)*

order to meet the Danish mathematicians. The trip was inspirational; he also met his future fiancée, Christine Kemp. When he returned to Oslo, Abel began work on the quintic equation once again, but this time, he attempted to prove that there was no radical expression for the solution. He was successful, and had his result published in French at his own expense. Sadly, there was no reaction from his intended audience—even CARL FRIEDRICH GAUSS was indifferent.

Abel's financial problems were complicated by his engagement to Kemp, but he managed to secure a small stipend to study languages in preparation for travel abroad. After this, he would receive a modest grant for two years of foreign study. In 1825 he departed with some friends for Berlin, and on his way through Copenhagen made the acquaintance of August Crelle, an influential engineer with a keen interest for mathematics. The two became lifelong friends, and Crelle agreed to start a German journal for the publication of pure mathematics. Many of Abel's papers were published in the first volumes, including an expanded version of his work on the quintic.

One of Abel's notable papers in *Crelle's Journal* generalized the binomial formula, which gives an expansion for the *n*th power of a binomial expression. Abel turned his thought toward infinite series, and was concerned that the sums had never been stringently determined. The result of his research was a classic paper on power series, with the determination of the sum of the binomial series for arbitrary exponents. Meanwhile, Abel failed to obtain a vacant position at the University of Sweden; his former teacher Holmboe was instead selected. It is noteworthy that Abel maintained his nobility of character throughout his frustrating life.

In spring 1826 Abel journeyed to Paris and presented a paper to the French Academy of Sciences that he considered his masterpiece: It treated the sum of integrals of a given algebraic function, and thereby generalized Euler's relation for elliptic integrals. This paper, over which Abel labored for many months but never published, was presented in October 1826, and AUGUSTIN-LOUIS CAUCHY and ADRIEN-MARIE LEGENDRE were appointed as referees. A report was not forthcoming, and was not issued until after Abel's death. It seems that Cauchy was to blame for the tardiness, and apparently lost the manuscript. Abel later rewrote the paper (neither was this work published), and the theorem described above came to be known as Abel's theorem.

After this disappointing stint in France, Abel returned to Berlin and there fell ill with his first attack of tuberculosis. Crelle assisted him with his illness, and tried to procure a position for him in Berlin, but Abel longed to return to Norway. Abel's new research transformed the theory of elliptic integrals to the theory of elliptic functions by using their inverses. Through this duality, elliptic functions became an important generalization of trigonometric functions. As a student in Oslo, Abel had already developed much of the theory, and this paper presented his thought in great detail.

Upon his return to Oslo in 1827, Abel had no prospects of a position, and managed to survive by tutoring schoolboys. In a few months Hansteen went on leave to Siberia and Abel became his substitute at the university. Meanwhile, Abel's work had started to stimulate interest among European mathematicians. In early 1828 Abel discovered that he had a young German competitor, CARL JACOBI, in the field of elliptic functions. Aware of the race at hand, Abel wrote a rapid succession of papers on elliptic functions and prepared a book-length memoir that would be published posthumously.

It seems that Abel had the priority of discovery over Jacobi in the area of elliptic functions; however, it is also known that Gauss was aware of the principles of elliptic functions long before either Abel or Jacobi, and had decided not to publish. At this time Abel started a correspondence with Legendre, who was also interested in elliptic functions. The mathematicians in France, along with Crelle, attempted to secure employment for Abel, and even petitioned the monarch of Sweden.

Abel's health was deteriorating, but he continued to write papers frantically. He spent summer 1828 with his fiancée, and when visiting her at Christmastime he became feverish due to exposure to the cold. As he prepared for his return to Oslo, Abel suffered a violent hemorrhage, and was confined to bed. At the age of 26 he died of tuberculosis on April 26, 1829; two days later, Crelle wrote him jubilantly that he had secured Abel an appointment in Berlin. In 1830 the French Academy of Sciences awarded its Grand Prix to Abel and Jacobi for their brilliant mathematical discoveries.

Abel became recognized as one of the greatest mathematicians after his death, and he truly accomplished much despite his short lifespan. The theory of elliptic functions would expand greatly during the later 19th century, and Abel's work contributed significantly to this development.

Further Reading

Bell, E. *Men of Mathematics*. New York: Simon and Schuster, 1965.

Ore, O. *Niels Henrik Abel, Mathematician Extraordinary*. Minneapolis: University of Minnesota Press, 1974.

Rosen, M. I. "Niels Henrik Abel and the Equation of the Fifth Degree," *American Mathematical Monthly* 102 (1995): 495–505.

Stander, D. "Makers of Modern Mathematics: Niels Henrik Abel," *Bulletin of the Institute of Mathematics and Its Applications* 23, nos. 6–7 (1987): 107–109.

Adelard of Bath

(unknown–ca. 1146)
British
Arithmetic

Little is known of the personal life of Adelard of Bath, but his work has been of great importance to the early revival of mathematics and natural philosophy during the medieval period. His translation of Greek and Arabic classics into Latin enabled the knowledge of earlier societies to be preserved and disseminated in Europe.

Adelard was a native of Bath, England, but his exact birth date is not known. He traveled widely in his life, first spending time in France, where he studied at Tours. For the next seven years he journeyed afar, visiting Salerno, Sicily, Cilicia, Syria, and perhaps even Palestine; it is thought that he also dwelt in Spain. His latter travels gave him an acquaintance with Arabic language and culture, though he may have learned Arabic while still in Sicily. By 1130 he had returned to Bath, and his writings from that time have some association with the royal court. One of his works, called *Astrolabe*, was apparently composed between 1142 and 1146; this is the latest recorded date of his activity.

Adelard made two contributions—*De eodem et diverso* (On sameness and diversity) and the *Questiones naturales* (Natural questions)—to medieval philosophy, written around 1116 and 1137, respectively. In *De eodem et diverso*, there is no evidence of Arabic influence, and he expresses the views of a quasi-Platonist. The *Questiones naturales* treats various topics in natural philosophy and shows the impact of his Arabic studies. Adelard's contribution to medieval science seems to lie chiefly in his translation of various works from Arabic.

His early endeavors in arithmetic, published in *Regule abaci* (By rule of the abacus), were quite traditional—his work reflected current arithmetical knowledge in Europe. These writings were doubtlessly composed prior to his familiarity with Arabic mathematics. Adelard also wrote on the topics of arithmetic, geometry, music, and astronomy. Here, the subject of Indian numerals and their basic operations is introduced as of fundamental importance.

Many scholars believe that Adelard was the first translator to present a full Latin version of EUCLID OF ALEXANDRIA's *Elements*. This began the process whereby the *Elements* would come to dominate late medieval mathematics; prior to Adelard's translation from the Arabic, there were only incomplete versions taken from the Greek. The first version was a verbatim transcription from the Arabic, whereas Adelard's second version replaces some of the proofs with instructions or summaries. This latter edition became the most popular, and was most commonly studied in schools. A third version appears to be a commentary and is attributed to Adelard; it enjoyed some popularity as well.

All the later mathematicians of Europe would read Euclid, either in Latin or Greek; indeed, this compendium of geometric knowledge would become a staple of mathematical education up to the present time. The Renaissance, and the consequent revival of mathematical discovery, was only made possible through the rediscovery of ancient classics and their translations. For his work as a translator and commentator, Adelard is remembered as an influential figure in the history of mathematics.

Further Reading

Burnett, C. *Adelard of Bath: An English Scientist and Arabist of the Early Twelfth Century*. London: Warburg Institute, University of London, 1987.

Agnesi, Maria Gaetana
(1718–1799)
Italian
Algebra, Analysis

Maria Gaetana Agnesi is known as a talented mathematician of the 18th century, and indeed was one of the first female mathematicians in the Western world. A mathematical prodigy with great linguistic talents, Agnesi made her greatest contribution through her clear exposition of algebra, geometry, and calculus; her colleagues acknowledged the value of her work within her own lifetime.

Born the eldest child of Pietro Agnesi and Anna Fortunato Brivio, Agnesi showed early interest in science. Her father, a wealthy professor of mathematics at the University of Bologna, encouraged and developed these interests. He established a cultural salon in his home, where his daughter would present and defend theses on a variety of scientific and philosophical topics. Some of the guests were foreigners, and Maria demonstrated her talent for languages by conversing with them in their own tongue; by age 11 she was familiar with Greek, German, Spanish, and Hebrew, having already mastered French by age five. At age nine she prepared a lengthy speech in Latin that promulgated higher education for women.

The topics of these theses, which were usually defended in Latin, included logic, ontology, mechanics, hydromechanics, elasticity, celestial mechanics and universal gravitation, chemistry, botany, zoology, and mineralogy. Her second published work, the *Propositiones philosophicae* (Propositions of philosophy, 1738), included almost 200 of these disputations. Agnesi's mathematical interests were developing at this time; at age 14 she was solving difficult problems in ballistics and analytic geometry. But after the publication of the *Propositiones philosophicae*, she decided to withdraw from her father's salon, since the social atmosphere was unappealing to her—in fact, she was eager to join a convent, but her father dissuaded her.

Nevertheless, Agnesi withdrew from the extroverted social life of her childhood, and devoted the next 10 years of her life to mathematics. After a decade of intense effort, she produced her *Instituzioni analitiche ad uso della gioventù italiana* (Analytical methods for the use of young Italians) in 1748. The two-volume work won immediate praise among mathematicians and brought Agnesi public acclaim. The objective of the thousand-page book was to present a complete and comprehensive treatment of algebra and analysis, including and emphasizing the new concepts of the 18th century. Of course, the development of differential and integral calculus was still in progress at this time; Agnesi would incorporate this contemporary mathematics into her treatment of analysis.

The material spanned elementary algebra and the classical theory of equations, coordinate geometry, the differential and integral calculus, infinite series, and the solution of elementary differential equations. Many of the methods and results were due solely to Agnesi, although her humble nature made her overly thorough in giving credit to her predecessors. Her name is often associated with a certain cubic curve called the *versiera* and known more commonly as the "witch of Agnesi." She was unaware that PIERRE DE FERMAT had studied the equation previously in 1665. This bell-shaped curve has many interesting properties and some applications in physics, and has been an ongoing source of fascination for many mathematicians.

Agnesi's treatise received wide acclaim for its excellent treatment and clear exposition. Translations into French and English from the original Italian were considered to be of great importance to the serious student of mathematics. Pope Benedict XIV sent her a congratulatory note in 1749, and in 1750 she was appointed to the chair of mathematics and natural philosophy at the University of Bologna.

However, Agnesi's reclusive and humble personality led her to accept the position only in honor, and she never actually taught at the university. After her father's death in 1752, she

Maria Agnesi studied the bell-shaped cubic curve called the *versiera,* which is more commonly known as the "witch of Agnesi." *(Courtesy of the Library of Congress)* began to withdraw from all scientific activity—she became more interested in religious studies and social work. She was particularly concerned with the poor, and looked after the education of her numerous younger brothers. By 1762 she was quite removed from mathematics, so that she declined the University of Turin's request that she act as referee for JOSEPH-LOUIS LAGRANGE's work on the calculus of variations. In 1771 Agnesi became the director of a Milanese home for the sick, a position she held until her death in 1799.

It is interesting to note that the sustained activity of her intellect over 10 years was able to produce the *Instituzioni*, a work of great excellence and quality. However, she lost all interest in mathematics soon afterward and made no further contributions to that discipline. Agnesi's primary contribution to mathematics is the *Instituzioni*, which helped to disseminate mathematical knowledge and train future generations of mathematicians.

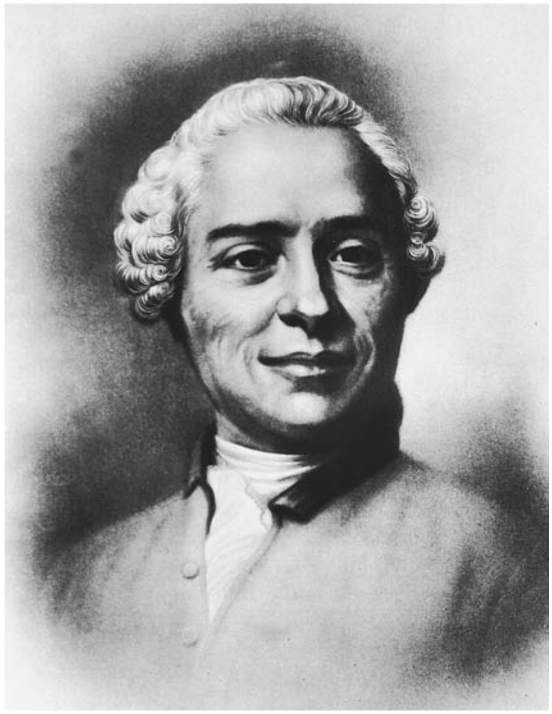
Further Reading

Grinstein, L., and P. Campbell. *Women of Mathematics*. New York: Greenwood Press, 1987.

Truesdell, C. "Correction and Additions for Maria Gaetana Agnesi," *Archive for History of Exact Science* 43 (1991): 385–386.

Alembert, Jean d′ (Jean Le Rond d'Alembert)
(1717–1783)
French
Mechanics, Calculus

In the wave of effort following SIR ISAAC NEWTON's pioneering work in mechanics, many mathematicians attempted to flesh out the mathematical aspects of the new science. Jean d'Alembert was noteworthy as one of these intellectuals, who contributed to astronomy, fluid mechanics, and calculus; he was one of the first persons to realize the importance of the limit in calculus.



Jean d'Alembert formulated several laws of motion, including d'Alembert's principle for decomposing constrained motions. *(Courtesy of the National Library of Medicine)*

Jean Le Rond d'Alembert was born in Paris on November 17, 1717. He was the illegitimate son of a famous salon hostess and a cavalry officer named Destouches-Canon. An artisan named Rousseau raised the young d'Alembert, but his father oversaw his education; he attended a Jansenist school, where he learned the classics, rhetoric, and mathematics.

D'Alembert decided on a career as a mathematician, and began communicating with the Académie des Sciences in 1739. During the next few years he wrote several papers treating the integration of differential equations. Although he had no formal training in higher mathematics, d'Alembert was familiar with the works of Newton, as well as the works of JAKOB BERNOULLI and JOHANN BERNOULLI.

In 1741 he was made a member of the Académie, and he concentrated his efforts on some problems in rational mechanics. The *Traité de dynamique* (Treatise on dynamics) was the fruit of his labor, a significant scientific work that formalized the new science of mechanics. The lengthy preface disclosed d'Alembert's philosophy of sensationalism (this idea states that sense perception, not reason, is the starting point for the acquisition of knowledge). He developed mechanics from the simple concepts of space and time, and avoided the notion of force. D'Alembert also presented his three laws of motion, which treated inertia, the parallelogram law of motion, and equilibrium. It is noteworthy that d'Alembert produced mathematical proofs for these laws.

The well-known d'Alembert's principle was also introduced in this work, which states that any constrained motion can be decomposed in terms of its inertial motion and a resisting (or constraining) force. He was careful not to overvalue the impact of mathematics on physics—he said that geometry's rigor was tied to its simplicity. Since reality was always more complicated than a mathematical abstraction, it is more difficult to establish truth.

In 1744 he produced a new volume called the *Traité de l'équilibre et du mouvement des fluides* (Treatise on the equilibrium and movement of fluids). In the 18th century a large amount of interest focused on fluid mechanics, since fluids were used to model heat, magnetism, and electricity. His treatment was different from that of DANIEL BERNOULLI, though the conclusions were similar. D'Alembert also examined the wave equation, considering string oscillation problems in 1747. Then in 1749 he turned toward celestial mechanics, publishing the *Recherches sur la précession des équinoxes et sur la nutation de l'axe de la terre* (*Research on the precession of the equinoxes and on the nodding of the earth's axis*), which treated the topic of the gradual change in the position of the earth's orbit.

Next, d'Alembert competed for a prize at the Prussian Academy, but blamed LEONHARD EULER for his failure to win. D'Alembert published his *Essai d'une nouvelle théorie de la résistance des fluides* (Essay on a new theory of the resistance of fluids) in 1752, in which the differential hydrodynamic equations were first expressed in terms of a field. The so-called hydrodynamic paradox was herein formulated—namely, that the flow before and behind an obstruction should be the same, resulting in the absence of any resistance. D'Alembert did not solve this problem, and was to some extent inhibited by his bias toward continuity; when discontinuities arose in the solutions of differential equations, he simply threw the solution away.

In the 1750s, interested in several nonscientific topics, d'Alembert became the science editor of the *Encyclopédie* (Encyclopedia). Later he wrote on the topics of music, law, and religion, presenting himself as an avid proponent of Enlightenment ideals, including a disdain for medieval thought.

Among his original contributions to mathematics, the ratio test for the convergence of an infinite series is noteworthy; d'Alembert viewed divergent series as nonsensical and disregarded them (this differs markedly from Euler's viewpoint). D'Alembert was virtually alone in his view of the derivative as the limit of a function, and his stress on the importance of continuity probably led him to this perspective. In the theory of probability d'Alembert was quite handicapped, being unable to accept standard solutions of gambling problems.

D'Alembert was known to be a charming, witty man. He never married, although he lived with his lover Julie de Lespinasse until her death in 1776. In 1772 he became the secretary of the Académie Française (the French Academy), and he increasingly turned toward humanitarian concerns. His later years were marked by bitterness and despair; he died in Paris on October 29, 1783.

Although he was well known as a preeminent scientist and philosopher, d'Alembert's mathematical achievements deserve special recognition. He greatly advanced the theory of mechanics in several of its branches, by contributing to its mathematical formulation and by consideration of several concrete problems.

Further Reading

Grimsley, R. *Jean d'Alembert, 1717–83*. Oxford: Clarendon Press, 1963.

Hankins, T. *Jean d'Alembert: Science and the Enlightenment*. Oxford: Clarendon Press, 1970.

Pappas, J. *Voltaire and d'Alembert*. Bloomington: Indiana University Press, 1962.

Wilson, C. "D'Alembert versus Euler on the Precession of the Equinoxes and the Mechanics of Rigid Bodies," *Archive for History of Exact Sciences* 37, no. 3 (1987): 233–273.

Apollonius of Perga

(ca. 262 B.C.E.–190 B.C.E.)
Greek
Geometry

Greek mathematics continued its development from the time of EUCLID OF ALEXANDRIA, and after ARCHIMEDES OF SYRACUSE one of the greatest mathematicians was Apollonius of Perga. He is mainly known for his contributions to the theory of conic sections (those plane figures obtained by slicing a cone at various angles). The fascination in this subject, revived in the 16th and 17th centuries, has continued into modern times with the onset of projective geometry.

Little information on his life has been preserved from the ravages of time, but it seems that Apollonius flourished sometime between the second half of the third century and the early second century B.C.E. Perga, a small Greek city in the southern portion of what is now Turkey, was his town of birth. Apollonius dwelt for some time in Alexandria, where he may have studied with the pupils of Euclid, and he later visited both Pergamum and Ephesus.

His most famous work, the *Conics*, was composed in the early second century B.C.E., and it soon became recognized as a classic text. Archimedes, who died around 212 B.C.E., appears to be the immediate mathematical predecessor of Apollonius, who developed many of the Syracusan's ideas. The *Conics* was originally divided into eight books, and had been intended as a treatise on conic sections. Before Apollonius's time, the basics of the theory of conic sections were known: Parabolas, hyperbolas, and ellipses could be obtained by appropriately slicing a cone with right, obtuse, or acute vertex angles, respectively. Apollonius employed an alternative method of construction that involved slicing a double cone at various angles, keeping the vertex angle fixed (this is the approach taken in modern times). This method had the advantage of making these curves accessible to the "application of areas," a geometrical formulation of quadratic equations that in modern time would be expressed algebraically. It is apparent that Apollonius's approach was refreshingly original, although the actual content of the *Conics* may have been well known. Much terminology, such as *parabola, hyperbola,* and *ellipse,* is due to Apollonius, and he generalizes the methods for generating sections.

The *Conics* contains much material that was already known, though the organization was according to Apollonius's method, which smoothly joined together numerous fragments of geometrical knowledge. Certain elementary results were omitted, and some few novel facts were included. Besides the material on the generation of sections, Apollonius described theorems on the rectangles contained by the segments of intersecting chords of a conic, the harmonic properties of pole and polar, properties of the focus, and the locus of three and four lines. He discusses the formation of a normal line to a conic, as well as certain inequalities of conjugate diameters. This work, compared with other Greek literature, is quite difficult to read, since the lack of modern notation makes the text burdensome, and the content itself is quite convoluted. Nevertheless, persistent study has rewarded many gifted mathematicians, including SIR ISAAC NEWTON, PIERRE DE FERMAT, and BLAISE PASCAL, who drew enormous inspiration from Apollonius's classic text.

In the work of PAPPUS OF ALEXANDRIA is contained a summary of Apollonius's other mathematical works: *Cutting off of a Ratio*, *Cutting off of an Area*, *Determinate Section*, *Tangencies*, *Inclinations*, and *Plane Loci*. These deal with various geometrical problems, and some of them involve the "application of an area." He uses the Greek method of analysis and synthesis: The problem in question is first presumed solved, and a more easily constructed condition is deduced from the solution ("analysis"); then from the latter construction, the original is developed ("synthesis"). It seems that Apollonius wrote still other documents, but no vestige of their content has survived to the present day. Apparently, he devised a number system for the representation of enormous quantities, similar to the notational system of Archimedes, though Apollonius generalized the idea. There are also references to the inscribing of the dodecahedron in the sphere, the study of the cylindrical helix, and a general treatise on the foundations of geometry.

So Apollonius was familiar with all aspects of Greek geometry, but he also contributed to the Euclidean theory of irrational numbers and derived approximations for pi more accurate than Archimedes'. His thought also penetrated the science of optics, where his deep knowledge of conics assisted the determination of various reflections caused by parabolic and spherical mirrors. Apollonius was renowned in his own time as a foremost astronomer, and he even earned the epithet of Epsilon, since the Greek letter of that name bears a resemblance in shape to the Moon. He calculates the distance of Earth to Moon as roughly 600,000 miles, and made various computations of the orbits of the planets. In fact, Apollonius is an important player in the development of geometrical models to explain planetary motion; HIPPARCHUS OF RHODES and CLAUDIUS PTOLEMY, improving upon his theories, arrived at the Ptolemaic system, a feat of the ancient world's scientific investigation possessed of sweeping grandeur and considerable longevity.

There was no immediate successor to Apollonius, though his *Conics* was recognized as a superb accomplishment. Various simple commentaries were produced, but interest declined after the fall of Rome, and only the first four books continued to be translated in Byzantium. Another three books of the *Conics* were translated into Arabic, and Islamic mathematicians remained intrigued by his work, though they made few advancements; the final (eighth) book has been lost. In the late 16th and early 17th centuries, several translations of Apollonius's *Conics* appeared in Europe and were voraciously studied by French mathematicians such as RENÉ DESCARTES, Pierre de Fermat, GIRARD DESARGUES, and Blaise Pascal. When Descartes propounded his analytic geometry, which took an algebraic, rather than constructive or geometrical, approach to curves and sections, interest in Apollonius's classic treatise began to wane. However, later in the 19th century, the *Conics* experienced a resurrection of curiosity with the introduction of projective geometry.

Further Reading

Heath, T. *Treatise on Conic Section*. New York: Barnes & Noble, 1961.

———. *A History of Greek Mathematics*. Oxford: Clarendon Press, 1921.

Hogendijk, J. "Arabic Traces of Lost Works of Apollonius," *Archive for History of Exact Sciences* 35, no. 3 (1986): 187–253.

Archimedes of Syracuse

(ca. 287 B.C.E.–212 B.C.E.)
Greek
Geometry, Mechanics

Of the mathematicians of Greek antiquity, Archimedes should be considered the greatest. His contributions to geometry and mechanics, as well as hydrostatics, place him on a higher pedestal than his contemporaries. And as his works were gradually translated and introduced into the West, he exerted as great an influence there as his thought already had in Byzantium and Arabia. In his method of exhaustion can be seen a classical predecessor of the integral calculus, which would be formally developed by BLAISE PASCAL, GOTTFRIED WILHELM VON LEIBNIZ, SIR ISAAC NEWTON, and others in the 17th century. His life story alone has inspired many mathematicians.

Archimedes is the great Greek mathematician who formulated the principles of hydromechanics and invented early techniques of integral calculus. *(Courtesy of the National Library of Medicine)*

As with many ancient persons, the exact details of Archimedes' life are difficult to ascertain, since there are several accounts of variable quality. His father was the astronomer Phidias, and it is possible that Archimedes was a kinsman of the tyrant of Syracuse, King Hieron II. Certainly he was intimate with the king, as his work *The Sandreckoner* was dedicated to Hieron's son Gelon. Born in Syracuse, Archimedes departed to Alexandria in order to pursue an education in mathematics; there he studied EUCLID OF ALEXANDRIA and assisted the development of Euclidean mathematics. But it was in Syracuse, where he soon returned, that he made most of his discoveries.

Although renowned for his contributions to mathematics, Archimedes also designed numerous mechanical inventions. The water snail, invented in Egypt to aid irrigation, was a screwlike contraption used to raise water. More impressive are the stories relating his construction and application of the compound pulley: Hieron had requested Archimedes to demonstrate how a small force could move a large weight. The mathematician attached a rope to a large merchant ship that was loaded with freight and passengers, and ran the line through a system of pulleys. In this manner, seated at a distance from the vessel, Archimedes was able to effortlessly draw the boat smoothly off the shore into the harbor.

Similar to the pulley, Archimedes discovered the usefulness of the lever, noting that the longer the distance from the fulcrum, the more weight the lever could move. Logically extending this principle, he asserted that it was feasible to move the world, given a sufficiently long lever. Another popular story relates that Hieron gave Archimedes the task of ascertaining whether a certain crown was made of pure gold, or whether it had been fraudulently alloyed with silver. As Archimedes pondered this puzzle, he came upon the bath, and noticed that the amount of water displaced was equal to the amount of his body that was immersed. This immediately put him in mind of a method to solve Hieron's problems, and he leapt out of the tub in joy, running naked toward his home, shrieking "Eureka!"

His skill in mechanical objects was unequaled, and Hieron often put him to use in improving the defenses of the city, insisting that Archimedes' intellect should be put to some practical application. When Marcellus and the Romans later came to attack Syracuse, they found the city impregnable due to the multiplicity of catapults, mechanical arms, burning mirrors, and various ballistic devices that Archimedes had built. Archimedes wrote a book entitled *On Spheremaking,* in which he describes how to construct a model planetarium designed to simulate the movement of Sun, Moon, and planets. It seems that Archimedes was familiar with Archytas's heliocentrism, and made use of this in his planetarium.

According to Plutarch, Archimedes was dedicated to pure theory and disdained the practical applications of mathematics to engineering; only those subjects free of any utility to society were considered worthy of wholehearted pursuit. Archimedes' mathematical works consist mainly of studies of area and volume, and the geometrical analysis of statics and hydrostatics. In computing the area or volume of various plane and solid figures, he makes use of the so-called Lemma of Archimedes and the "method of exhaustion." This lemma states that the difference of two unequal magnitudes can be formed into a ratio with any similar magnitude; thus, the difference of two lines will always be a line and not a point. The method of exhaustion involves subtracting a quantity larger than half of a given magnitude indefinitely, and points to the idea of the eternal divisibility of the continuum (that one can always take away half of a number and still have something left). These ideas border on notions of the infinitesimal—the infinitely small—and the idea of a limit, which are key ingredients of integral calculus; however, the Greeks were averse to the notion of infinity and infinitesimals, and Archimedes shied away from doing anything that he felt would be regarded as absurd.

The method of exhaustion, which was used rarely in Euclid's *Elements,* will be illustrated through the following example: In *On the Measurement of the Circle,* Archimedes assumes, for the sake of contradiction, that the area of a right triangle with base equal to the circumference and height equal to the radius of the circle is actually greater than the area of the circle. Then he is able, using the Lemma of Archimedes, to inscribe a polygon in the circle, with the same area as the triangle; this contradiction shows that the area of the triangle cannot be greater than the circle, and he makes a similar argument that it cannot be less.

The basic concept of the method of approximation, which is similar to the method of exhaustion, is to inscribe regular figures within a given plane figure and solid such that the remaining area or volume is steadily reduced; the area or volume of the regular figures can be easily calculated, and this will be an increasingly accurate approximation. The remaining area or volume is "exhausted." Of course, the modern way to obtain an exact determination of measure is via the limit; Archimedes avoided this issue by demonstrating that the remaining area or volume could be made as small as desired by inscribing more regular figures. Of course, one could perform the same procedure with circumscribing regular figures.

He also applied these methods to solids, computing the surface area and volume of the sphere, and the volume of cones and pyramids. Archimedes' methods were sometimes purely geometrical, but at times used principles from statics, such as a "balancing method." His knowledge of the law of the lever and the center of gravity for the triangle, together with his approximation and exhaustion methods, enabled him to improve the proofs of known theorems as well as establish completely new results.

Archimedes also made some contributions in the realm of numerical calculations, producing some highly accurate approximations for pi and the square root of three. In *The Sandreckoner* he devises a notation for enormous numbers and estimates the number of grains of sand to fill the universe. In *On the Equilibrium of Planes* he proves the law of the lever from geometrical principles, and in *On Floating Bodies* he explains the concept of hydrostatic pressure. The so-called Principle of Archimedes states that solids placed in a fluid will be lighter in the fluid by an amount equal to the weight of the fluid displaced.

His influence on later mathematics was extensive, although Archimedes may not have enjoyed much fame in his own lifetime. Later Greeks, including PAPPUS OF ALEXANDRIA and Theon of Alexandria, wrote commentaries on his writings, and later still, Byzantine authors studied his work. From Byzantium his texts came into the West before the start of the Renaissance; meanwhile, Arabic mathematicians were familiar with Archimedes, and they exploited his methods in their own researches into conic sections. In the 12th century translations from Arabic into Latin appeared, which LEONARDO FIBONACCI made use of in the 13th century. By the 1400s knowledge of Archimedes had expanded throughout parts of Europe, and his mathematics later influenced SIMON STEVIN, Johannes Kepler, GALILEO GALILEI, and BONAVENTURA CAVALIERI.

Perhaps the best-known story concerning Archimedes relates his death, which occurred in 212 B.C.E. during the siege of Syracuse by the Romans. Apparently, he was not concerned with the civic situation, and was busily making sand diagrams in his home (at this time he was at least 75 years old). Although the Roman general Marcellus had given strict orders that the famous Sicilian mathematician was not to be harmed, a Roman soldier broke into Archimedes' house and spoiled his diagram. When the aged mathematician vocally expressed his displeasure, the soldier promptly slew him.

Archimedes was an outstanding mathematician and scientist. Indeed, he is considered by many to be one of the greatest three mathematicians of all time, along with CARL FRIEDRICH GAUSS and Newton. Once discovered by medieval Europeans, his works propelled the discovery of calculus. It is interesting that this profound intellect was remote in time and space from the great classical Greek mathematicians; Archimedes worked on the island of Syracuse, far from Athens, the source of much Greek thought, and he worked centuries after the decline of the Greek culture.

Further Reading

Aaboe, A. *Episodes from the Early History of Mathematics*. Washington, D.C.: Mathematical Association of America, 1964.

Brumbaugh, R. *The Philosophers of Greece*. New York: Crowell, 1964.

Dijksterhuis, E. *Archimedes*. Copenhagen: E. Munksgaard, 1956.

Heath, T. *A History of Greek Mathematics*. Oxford: Clarendon Press, 1921.

Hollingdale, S. "Archimedes of Syracuse: A Tribute on the 22nd Century of His Death," *Bulletin of the Institute of Mathematics and Its Applications* 25, no. 9 (1989): 217–225.

Laird, W. "Archimedes among the Humanists," *Isis* 82, no. 314 (1991): 629–638.

Neugebauer, O. "Archimedes and Aristarchus," *Isis* 34 (1942): 4–6.

Osborne, C. "Archimedes on the Dimension of the Cosmos," *Isis* 74, no. 272 (1983): 234–242.

Aristarchus of Samos

(ca. 310 B.C.E.–230 B.C.E.)
Greek
Trigonometry

Renowned as the first person to propose a heliocentric theory (that the planets revolve around the Sun) of the solar system, Aristarchus was both an important astronomer and a first-rate mathematician. Little is known of his life, but his works have survived, in which he calculates various astronomical distances millennia before the invention of modern telescopes.

Apparently, Aristarchus was born on the island of Samos, which lies in the Aegean Sea close to the city of Miletus, a center for science and learning in the Ionian civilization. He studied under Strato of Lampsacos, director of the Lyceum founded by Aristotle. It is thought that Aristarchus was taught by Strato in Alexandria rather than Athens. His approximate dates are determined by the records of CLAUDIUS PTOLEMY and ARCHIMEDES OF SYRACUSE. Aristarchus's only work still in existence is his treatise *On the Sizes and Distances of the Sun and Moon*.

Among his peers, Aristarchus was known as "the mathematician," which may have been merely descriptive. At that time, the discipline of astronomy was considered part of mathematics, and Aristarchus's *On Sizes and Distances* primarily treats astronomical calculations. According to Vitruvius, a Roman architect, Aristarchus was an expert in all branches of mathematics, and was the inventor of a popular sundial consisting of a hemispherical bowl with a vertical needle poised in the center. It seems that his discoveries in *On Sizes and Distances* of the vast scale of the universe fostered an interest in the physical orientation of the solar system, eventually leading to his heliocentric conception of the Sun in the center.

Heliocentrism has its roots in the early Pythagoreans, a religious/philosophical cult that thrived in the fifth century B.C.E. in southern Italy. Philolaus (ca. 440 B.C.E.) is attributed with the idea that the Earth, Moon, Sun, and planets orbited around a central "hearth of the universe." Hicetas, a contemporary of Philolaus, believed in the axial rotation of the Earth. The ancient historians credit Heraclides of Pontus (ca. 340 B.C.E.) with the Earth's rotation about the Sun, but Aristarchus is said to be the first to develop a complete heliocentric theory: The Earth orbits the Sun while at the same time spinning about its axis.

It is interesting that the heliocentric theory did not catch on. The idea did not attract much attention, and the philosophical speculations of the Ionian era were already waning, to be replaced by the increasingly mathematical feats of APOLLONIUS OF PERGA, HIPPARCHUS OF RHODES, and Ptolemy. Due to trends in intellectual and religious circles, geocentrism became increasingly popular. Not until Nicolaus Copernicus, who lived 18 centuries later, resurrected Aristarchus's hypothesis did opinion turn away from considering the Earth as the center of the universe.

Living after EUCLID OF ALEXANDRIA and before Archimedes, Aristarchus was able to produce rigorous arguments and geometrical constructions, a distinguishing characteristic of the better mathematicians. The attempt to make various measurements of the solar system without a telescope seems incredible, but it involved the simple geometry of triangles. With the Sun (S), Earth (E), and Moon (M) as the three vertices of a triangle, the angle EMS will be a right angle when the Moon is exactly half in shadow. Through careful observation, it is possible to measure the angle MES, and thus the third angle ESM can be deduced. Once these angles are known, the ratio of the length of the legs, that is, the distance to the Moon and the distance to the Sun, can be determined. Of course, this procedure is fraught with difficulties, and any slight error in estimating the angles will throw off the whole calculation. Aristarchus estimated angle MES to be approximately 87 degrees, when it is actually 89 degrees and 50 minutes. From this, he deduces that the distance to the Sun is about 20 times greater than the distance to the Moon, when in actuality it is 400 times greater. His theory was sound, but Aristarchus was inhibited by his crude equipment.

This is discussed in *On Sizes and Distances*, where he states several assumptions and from these proves the above estimate on the distance to the Sun and also states that the diameter of Sun and Moon are related in the same manner (the Sun is about 20 times as wide across as the Moon). He also computes that the ratio of the diameter of the Sun to the diameter of the Earth is between 19:3 and 43:6, an underestimate.

It is noteworthy that trigonometry had not yet been developed, and yet Aristarchus developed methods that essentially estimated the sines of small angles. Without precise means of calculation, Aristarchus was unable to attain accurate results, although his method was brilliant. Because heliocentrism was not accepted at the time, Aristarchus failed to achieve much fame in his own lifetime. Nevertheless, he was one of the first mathematicians to obtain highly accurate astronomical measurements.

Further Reading

Heath, T. *Aristarchus of Samos, the Ancient Copernicus*. Oxford: Clarendon Press, 1966.

———. *A History of Greek Mathematics*. Oxford: Clarendon Press, 1921.

Neugebauer, O. *A History of Ancient Mathematical Astronomy*. New York: Springer-Verlag, 1975.

———. "Archimedes and Aristarchus," *Isis* 34 (1942): 4–6.

Aryabhata I

(476–550)
Indian
Algebra, Geometry

Little is known of the life of Aryabhata, who is called Aryabhata I in order to distinguish him from another mathematician of the same name who lived four centuries later. Aryabhata played a role in the development of the modern current number system and made contributions to number theory at a time when much of Europe was enveloped in ignorance.

He was born in India and had a connection with the city Kusumapura, the capital of the Guptas during the fourth and fifth centuries; this place is thought to be the city of his birth. Certainly, his *Aryabhatiya* was written in Kusumapura, which later became a center of mathematical learning.

Aryabhata wrote two works: the *Aryabhatiya* in 499, when he was 23 years old, and another treatise, which has been lost. The former work is a short summary of Hindu mathematics, consisting of three sections on mathematics, time and planetary models, and the sphere. The sections on mathematics contain 66 mathematical rules without proof, dealing with arithmetic, algebra, plane trigonometry, and spherical trigonometry. However, it also contains more advanced knowledge, such as continued fractions, quadratic equations, infinite series, and a table of sines. In 800 this work was translated into Arabic, and had numerous Indian commentators.

Aryabhata's number system, the one he used in his book, gives a number for each of the 33 letters of the Indian alphabet, representing the first 25 numbers as well as 30, 40, 50, 60, 70, 80, 90, and 100. It is noteworthy that he was familiar with a place-value system, so that very large numbers could easily be described and manipulated using this alphabetical notation. Indeed, it seems likely that Aryabhata was familiar with zero as a placeholder. The Indian place-value number system, which would later greatly influence the construction of the modern system, facilitated calculations that would be infeasible under more primitive models, such as Roman numerals. Aryabhata appears to be the originator of this place-value system.

In his examination of algebra, Aryabhata first investigates linear equations with integer coefficients—apparently, the *Aryabhatiya* is the first written work to do so. The question arose from certain problems of astronomy, such as the computation of the period of the planets. The technique is called *kuttaka*, which means "to pulverize," and consists of breaking the equation into related problems with smaller coefficients; the method is similar to the Euclidean algorithm for finding the greatest common divisor, but is also related to the theory of continued fractions.

In addition, Aryabhata gave a value for pi that was accurate to eight decimal places, improving on ARCHIMEDES OF SYRACUSE's and APOLLONIUS OF PERGA's approximations. Scholars have argued that he obtained this independently of the Greeks, having some particular method for approximating pi, but it is not known exactly how he did it; Aryabhata also realized that pi was an irrational number. His table of sines gives approximate values at intervals of less than four degrees, and uses a trigonometric formula to accomplish this.

Aryabhata also discusses rules for summing the first *n* integers, the first *n* squares, and the first *n* cubes; he gives formulas for the area of triangles and of circles. His results for the volumes of a sphere and of a pyramid are incorrect, but this may have been due to a translation error. Of course, these latter results were well known to the Greeks and might have come to Aryabhata through the Arabs.

As far as the astronomy present in the text, which the mathematics is designed to elucidate, there are several interesting results. Aryabhata gives an excellent approximation to the circumference of the Earth (62,832 miles), and explains the rotation of the heavens through a theory of the axial rotation of the Earth. Ironically, this (correct) theory was thought ludicrous by later commentators, who altered the text in order to remedy Aryabhata's mistakes. Equally remarkable is his description of the planetary orbits as ellipses—only highly accurate astronomical data provided by superior telescopes allowed European astronomers to differentiate between circular and elliptical orbits. Aryahbhata gives a correct explanation of the solar and lunar eclipses, and attributes the light of the Moon to reflected sunlight.

Aryabhata was of great influence to later Indian mathematicians and astronomers. Perhaps most relevant for the later development of mathematics was his place-number system. His theories were exceedingly advanced considering the time in which he lived, and the accurate computations of astronomical measurements illustrated the power of his number system.

Further Reading

Gupta, R. C. "Aryabhata, Ancient India's Great Astronomer and Mathematician," *Mathematical Education* 10, no. 4 (1976): B69–B73.

———. "A Preliminary Bibliography on Aryabhata I," *Mathematical Education* 10, no. 2 (1976): B21–B26.

Ifrah, G. *A Universal History of Numbers: From Prehistory to the Invention of the Computer*. New York: John Wiley, 2000.

van der Waerden, B. "The 'Day of Brahman' in the Work of Aryabhata," *Archive for History of Exact Sciences* 38, no. 1 (1988): 13–22.

B

Babbage, Charles

(1792–1871)
British
Analysis

The name of Charles Babbage is associated with the early computer. Living during the industrial age, in a time when there was unbridled optimism in the potential of machinery to improve civilization, Babbage was an advocate of mechanistic progress, and spent much of his lifetime pursuing the invention of an "analytic engine." Although his ambitious project eventually ended in failure, his ideas were important to the subsequent develop of computer logic and technology.

Born on December 26, 1792, in Teignmouth, England, to affluent parents, Babbage exhibited great curiosity for how things worked. He was educated privately by his parents, and by the time he registered at Cambridge in 1810, he was far ahead of his peers. In fact, it seems that he knew more than even his teachers, as mathematics in England had lagged far behind the rest of Europe. Along with George Peacock and John Herschel, he campaigned vigorously for the resuscitation of English mathematics. Together with Peacock and Herschel, he translated Lacroix's *Differential and Integral Calculus*, and became an ardent proponent of GOTTFRIED WILHELM VON LEIBNIZ's notation over SIR ISAAC NEWTON's.

Upon graduating, Babbage became involved in many diverse activities: He wrote several papers on the theory of functions and applied mathematics and helped to found several progressive learned societies, such as the Astronomical Society in 1820, the British Association in 1831, and the Statistical Society of London in 1834. He was recognized for his excellent contributions to mathematics, being made a fellow of the Royal Society in 1816 and Lucasian professor of mathematics at Cambridge in 1827; he held this latter position for 12 years without teaching, because he was becoming increasingly absorbed by the topic of mechanizing computation.

Babbage viewed science as an essential part of civilization and culture, and even thought that it was the government's responsibility to encourage and advance science by offering grants and prizes. Although this viewpoint is fairly common today, Babbage was one of its first advocates; before his time, much of science and mathematics was conducted in private research by men of leisure. He also advocated pedagogical reform, realizing that great teaching was crucial for the future development of mathematics; however, he did little with his chair at Cambridge toward realizing this goal.

His interests were remarkably diverse, including probability, cryptanalysis, geophysics, astronomy, altimetry, ophthalmoscopy, statistical

Charles Babbage, inventor of an early mechanical computer and founder of computer science *(Courtesy of the Library of Congress)*

linguistics, meteorology, actuarial science, lighthouse technology, and climatology. Babbage devised a convenient notation that simplified the drawing and reading of engineering charts. His literature on operational research, concerned with mass production in the context of pin manufacture, the post office, and the printing trade, has been especially influential.

Babbage was, as a young man, lively and sociable, but his growing obsession with constructing computational aids made him bitter and grumpy. Once he realized the extent of errors in existing mathematical tables, his mind turned to the task of using machinery to accomplish faultless calculations. Initially, he imagined a steam-powered calculator for the computation of trigonometric quantities; he began to envision a machine that would calculate functions and also print out the results.

The theory behind his machine was the method of finite differences—a discrete analog of the continuous differential calculus. Any polynomial of *n*th degree can be reduced, through successive differences, to a constant; the inverse of this procedure, taking successive sums, would be capable of computing the values of a polynomial, given some initial conditions. In addition, this concept could be extended to most nonrational functions, including logarithms; this would allow the mechanistic computation of the value of an arbitrary function.

Unfortunately, Babbage did not succeed. He continually thought up improvements for the system, becoming more ambitious for the final "Difference Engine Number One." This machine would handle sixth-order differences and 20 decimal numbers—a goal more grandiose than feasible. He never completed the project, though a Swedish engineer, in Babbage's own lifetime, built a modest working version based on a magazine account of the Englishman's dream. It seems that the principal reason for Babbage's failure was the prohibitive cost, though another cause is found in his new design to build an "analytical engine."

The analytical engine, in its design and planning, was a forerunner of the modern computer. Based on Joseph-Marie Jacquard's punch cards used in weaving machinery, Babbage's machine would be run by inserting cards with small holes; springy wires would move through the holes to operate certain levers. This concept described a machine of great versatility and power. The mill, the center of the machine, was to possess 1,000 columns with 50 geared wheels apiece: up to 1,000 50-digit numbers could be operated on with one of the four main arithmetic operations. Data, operation, and function cards could be inserted to provide information on variables, programs, and constants to the mill. The output would be printed, and another part of the machine would check for errors, store information, and make decisions. This corresponds to the memory and logic flow components of a modern computer. However, in one important aspect Babbage's analytical engine differs from the digital computer: His was based on a decimal system, whereas computers operate on a binary system.

Although the plans for this machine impressed all who viewed them, Babbage did not receive any financial support for its construction. He died on October 18, 1871, in London, without seeing the completion of his mechanistic projects. However, his son later built a small mill and printer, which is kept in the Science Museum of London.

Babbage was a highly creative mathematician whose ideas foreshadowed the major thrust of computer science in the second half of the 20th century. His work in pure mathematics has had little impact on successive generations of mathematicians, but his ideas on the analytical engine would be revisited over the next century, culminating in the design of early computers in the mid-1900s.

Further Reading

Babbage, H. *Babbage's Calculating Engines*. Los Angeles: Tomash, 1982.

Buxton, H. *Memoir of the Life and Labors of the Late Charles Babbage Esq., F.R.S.* Los Angeles: Tomash, 1988.

Dubbey, J. *The Mathematical Work of Charles Babbage*. Cambridge: Cambridge University Press, 1978.

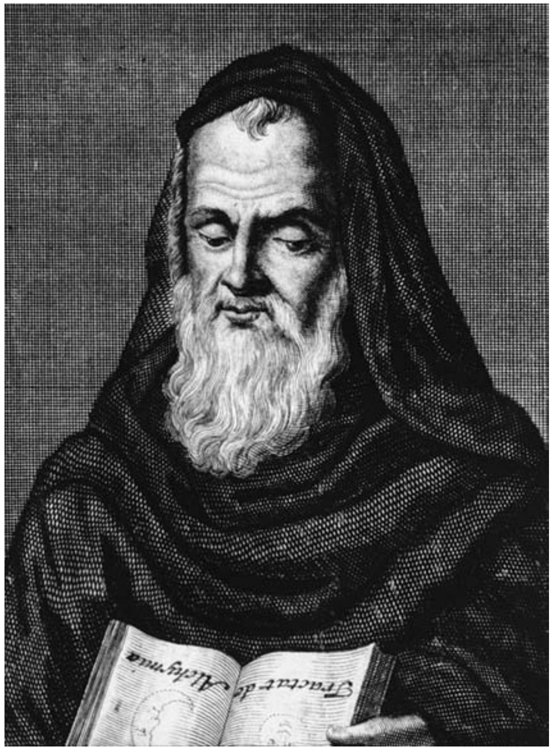
Hyman, A. *Charles Babbage: Pioneer of the Computer*. Princeton, N.J.: Princeton University Press, 1982.

Morrison, P., and E. Morrison. *Charles Babbage and His Calculating Engines*. New York: Dover Publications, 1961.

Wilkes, M. "Babbage as a Computer Pioneer," *Historia Mathematica* 4, no. 4 (1977): 415–440.

Bacon, Roger

(ca. 1214–ca. 1292)
British
Arithmetic



Roger Bacon proposed that mathematical knowledge should be arrived at through reason rather than authority. *(Courtesy of the National Library of Medicine)*

In 13th-century Europe, there was no pursuit of science as there is today: the medieval church, having gone so far as to make reason irrelevant in matters of faith and knowledge, substituting the unmitigated authority of papal decree and canon law, reigned over a stifling intellectual climate. However, the use of reason and empiricism, when coupled with the knowledge of a rational God's creation of a rational world, would prove to be the epistemology of science for the next several centuries, which resulted in numerous discoveries. Roger Bacon was an early figure in this paradigm shift, vigorously acting as a key proponent of the utility of mathematics and logic within the spheres of human knowledge. Natural philosophy, which in his view was subservient to theology, could serve toward the advancement of the human task generally speaking (the dominion and ordering of the Earth and, more specifically, the development of the church). Later scientific endeavor, starting in the 18th and 19th centuries, would abandon these theistic roots in favor of reason as the sole authority in man's pedagogical quest; but Bacon's promotion of the use of mathematics in partnership with faith in God was to remain the guiding epistemology for several centuries.

Bacon's birth has been calculated to be approximately 1214, though scholars differ on this detail since there is no exact record. This Englishman came of a family that had suffered persecution from the baronial party, due to their failed support of Henry III. His early instruction in the Latin classics, including Seneca and Cicero, led to his lifelong fascination with natural philosophy and mathematics, further inculcated at Oxford. After receiving his M.A. degree in about 1240, he apparently lectured in the Faculty of the Arts at Paris from 1241 to 1246. He discussed various topics from Aristotle's works, and he was a vehement advocate of complete instruction in foreign languages. Bacon underwent a drastic change in his conception of knowledge after reading the works of Robert Grosseteste (a leading philosopher and mathematician of the region) when he returned to Oxford in 1247; he invested considerable sums of money for experimental equipment, instruments, and books, and sought out acquaintance with various learned persons. Under Grosseteste's influence, Bacon developed the belief that languages, optics, and mathematics were the most important scientific subjects, a view he maintained his whole life.

By 1251 he had returned to Paris, and he entered the Franciscan order in 1257. The chapter of Narbonne was presided over by Bonaventure, who was opposed to inquiries not directly related to theology; he disagreed sharply with Bacon on the topics of alchemy and astrology, which he viewed as a complete waste of time. Bacon, on the other hand, while agreeing that they had no discernible or predictable impact on the fates of individuals, thought it possible for the stars to exert a generic influence over the affairs of the world; he also experimented in alchemy, the quest to transmute lead into gold. Due to these political difficulties, Bacon made various proposals on education and science to Cardinal Guy de Folques, who was soon elected Pope Clement IV in 1265. As pope he formally requested Bacon to submit his philosophical writings, and the Englishman soon produced three famous works: *Opus maius* (Great work), *Opus minus* (Smaller work), and *Opus tertium* (Third work) within the next few years.

The *Opus maius* treated his opinions on natural philosophy and educational reform. Authority and custom were identified as impediments to learning; although Bacon submitted to the authority of the Holy Scriptures, he believed the wisdom contained therein needed to be developed by reason, rightly informed by faith. In this one sees some early seeds of Protestant thought about the proper balance of authority and reason. However, Bacon was not a believer in pure deduction detached from the observed world, like the Greek philosophers and mathematicians of antiquity; rather, he argued for requisition of experience. Information obtained through the exterior senses could be measured and quantified through instruments and experimental devices and analyzed through the implementation of mathematics. By studying the natural world, it was possible, Bacon argued, to arrive at some understanding of the Creator of that natural world. Thus, all of human knowledge was conceived in a harmonious unity, guided and led by theology as the regent of science. Hence it was necessary to deepen the understanding of languages, mathematics, optics, experimental science, alchemy, metaphysics, and moral philosophy.

Bacon's view on authority was somewhat progressive: without moderation, authority would prevent the plowing of intellectual furrows given provenience by rational disputation.

However, it must not be thought that a predecessor of nihilism, moral relativism, or other antiauthoritative systems can be found in Bacon—he believed in one truth (Christianity), but sought to use reason as a fit tool for advancing the interests of the kingdom of God and the civilization of man. The heathen should be converted by argument and persuasion, never by force.

Mathematics was to play an important role in Bacon's entire system. Of course, he understood the term in a broad sense, as inclusive of astronomy and astrology, optics, physical causation, and calendar reform, with even applications to purely religious matters. His work in optics relied heavily on geometry, and stood on the shoulders of EUCLID OF ALEXANDRIA, CLAUDIUS PTOLEMY, and ABU ALI AL-HAYTHAM, as well as Grosseteste. Along with Grosseteste, he advocated the use of lenses for incendiary and visual purposes. Bacon's ideas on refraction and reflection constituted a wholly new law of nature. His work on experimental science laid down three main goals: to certify deductive reasoning from other subjects, such as mathematics, by experimental observation; to add new knowledge not attainable by deduction; and to probe the secrets of nature through new sciences. The last prerogative can be seen as an effort toward attaining practical magic—the requisitioning of nature toward spectacular and utilitarian ends.

Bacon lists four realms of mathematical activity: human business, divine affairs (such as chronology, arithmetic, music), ecclesiastical tasks (such as the certification of faith and repair of the calendar), and state works (including astrology and geography). Mathematics, the "alphabet of philosophy," had no limits to its range of applicability, although experience was still necessary in Bacon's epistemology. Despite his glowing praise of "the door and key of the sciences," it appears that Bacon's facility in mathematics was not great. Although he has some original results in engineering, optics, and astronomy, he does not furnish any proofs or theorems of his own devising.

He also made some contributions in the areas of geography and calendar reform. He stated the possibility of journeying from Spain to India, which may have influenced Columbus centuries later. Bacon's figures on the radius of the Earth and ratio of land and sea were fairly accurate, but based on a careful selection of ancient authorities. His map of the known world, now lost, seems to have included lines of latitude and longitude, with the positions of famous towns and cities. Bacon discussed the errors of the Julian calendar with great perspicuity, and recommended the removal of one day in 125 years, similar to the Gregorian system.

Certainly, after his death, Bacon had many admirers and followers in the subsequent centuries. He continued writing various communications on his scientific theories, but sometime after 1277 he was condemned and imprisoned in Paris by his own Franciscan order, possibly for violating a censure. His last known writing was published in 1292, and he died sometime afterward.

Bacon contributed generally to the advance of reason and a rational approach to knowledge in Europe; his efforts influenced not only the course of mathematics but also the history of science more generally. The writings of Bacon would be familiar to later generations of mathematicians working in the early 17th century.

Further Reading

Bridges, J. *The Life and Work of Roger Bacon: An Introduction to the Opus Majus*. Merrick, N.Y.: Richwood Publishing Company, 1976.

Easton, S. *Roger Bacon and His Search for a Universal Science; a Reconsideration of the Life and Work of Roger Bacon in the Light of His Own Stated Purposes*. Westport, Conn.: Greenwood Press, 1970.

Lindberg, D. *Roger Bacon's Philosophy of Nature: A Critical Edition*. Oxford: Clarendon Press, 1983.

———. "Science As Handmaiden: Roger Bacon and the Patristic Tradition," *Isis* 78, no. 294 (1987): 518–536.

Baire, René-Louis
(1874–1932)
French
Analysis

In the late 19th century some of the ideas on the limits of sequences of functions were still vague and ill formulated. René Baire greatly advanced the theory of functions by considering issues of continuity and limit; his efforts helped to solidify the intuitive notions then in circulation.

René-Louis Baire was born in Paris on January 21, 1874, one of three children in a middle-class family. His parents endured hardship in order to send Baire to school, but he won a scholarship in 1886 that allowed him to enter the Lycée Lakanal. He completed his studies with high marks and entered the École Normale Supérieure in 1892.

During his next three years, Baire became one of the leading students in mathematics, earning first place in his written examination. He was a quiet, introspective young man of delicate health, which would plague him throughout his life. In the course of his oral presentation of exponential functions, Baire realized that the demonstration of continuity that he had learned was insufficient; this realization led him to study the continuity of functions more intensely and to investigate the general nature of functions.

In 1899 Baire defended his doctoral thesis, which was concerned with the properties of limits of sequences of continuous functions. He embarked on a teaching career at local lycées, but found the schedule too demanding; eventually he obtained an appointment as professor of analysis at the Faculty of Science in Dijon in 1905. Meanwhile, Baire had already written some papers on discontinuities of functions, and had also suffered a serious illness involving the constriction of his esophagus. In 1908 he completed a major treatise on mathematical analysis that breathed new life into that subject. From 1909 to 1914 his health was in continual decline, and Baire struggled to fulfill his teaching duties; in 1914 he obtained a leave of absence and departed for Lausanne. Unfortunately, the eruption of war prevented his return, and he was forced to remain there in difficult financial circumstances for the next four years.

His mathematical contributions were mainly focused around the analysis of functions. Baire developed the concept of semicontinuity, and perceived that limits and continuity of functions had to be treated more carefully than they had been. His use of the transfinite number exercised great influence on the French school of mathematics over the next several decades. Baire's most lasting contributions are concerned with the limits of continuous functions, which he divided into various categories. He provided the proper framework for studying the theory of functions of a real variable; previously, interest was peripheral, as mathematicians were only interested in real functions that came up in the course of some other investigation. Thus, Baire effected a reorientation of thought.

Baire's illness made him incapable of resuming his grand project, and after the war he focused instead on calendar reform. He later received the ribbon of the Legion of Honor and was elected to the Academy of Sciences; sadly, his last years were characterized by pain and financial struggles. As a result, he was able to devote only limited amounts of time to mathematical research. He died in Chambéry, France, on July 5, 1932.

Baire's work played an important role in the history of modern mathematics, as it represents a significant step in the maturation of thought. His ideas were highly regarded by ÉMILE BOREL and HENRI LEBESGUE, and exerted much influence on subsequent French and foreign mathematicians.