ROUTLEDGE ADVANCES IN RISK MANAGEMENT

Emerging Financial Derivatives

Understanding exotic options and structured products

Jerome Yen and Kin Keung Lai

ROUTLEDGE

Emerging Financial Derivatives

Exotic options and structured products have been two of the most popular financial products over the past ten years and will soon become very important to the emerging markets, especially China. This book first discusses the products' recent development in the world and provides a comprehensive overview of the major products. The book also discusses the risks of issuing and buying such products as well as the techniques to price them and to assess the risks. Volatility is the most important factor in determining the return and risk. Therefore, a significant part of the book's content discusses how we can measure the volatility by using local and stochastic volatility models — the Heston Model and Dupire Model, the volatility surface, the term structure of volatility, variance swaps, and breakeven volatility.

The book introduces a set of dimensions which can be used to describe structured products to help readers to classify them. It also describes the more commonly traded exotic options with details. The book discusses key features of each exotic option which can be used to develop structured products and covers their pricing models and when to issue such products that contain such exotic options. This book contains several case studies about how to use the models or techniques to price and hedge risks. These case analyses are illuminating.

Jerome Yen is currently a professor of the College of Business at Tung Wah College, Hong Kong and a visiting professor in the Department of Finance at Hong Kong University of Science and Technology. He is also the director of HKUST's Quantitative Finance program where over 70 percent of graduates went to investment banks like Goldman Sachs and Morgan Stanley. He received his Ph.D. in 1992 in Systems Engineering and Management Information Systems from the University of Arizona, USA.

Kin Keung Lai received his Ph.D. at Michigan State University, USA. He is currently the Chair Professor of Management Science at the City University of Hong Kong. He is also the Director of the Invesco-Great Wall Research Unit on Risk Analysis and Business Intelligence (RABI) at the College of Business. Prior to his current post, he was a Senior Operational Research Analyst for Cathay Pacific Airways and an Area Manager on Marketing Information Systems for Union Carbide Eastern.

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Preface

Over the past decade, structured products experienced dramatic changes in their nature and in the appetite of investors. Although the demand for sophisticated products decreased in the Western world, the conditions in emerging markets such as China and India were different. The size of the investment product markets, which includes structured products, in China increased by over 30 percent in 2012, and we expect a similar number in 2014. Therefore, there is still an increasing need for structure developers who know how to develop structured products with underlying assets that cover equities/stocks, foreign exchange, and commodities. Also, for the emerging market, a great escalation in the sophistication of such structured products has taken place to meet the different requirements of investors to cope with the markets under different conditions.

Structured products can be divided into two categories: (1) flow products and (2) theme-based or ad hoc products. Flow products are standard products such as equity-linked notes (ELNs) that are always available on the product shelf. Theme-based or ad hoc products are designed to take advantage of special market conditions, for example, gold price bullish movement, so that investors can earn higher returns. Therefore, we feel that the structured products business is not just technical driven; two very important components are forecasting the potential movement of the underlying asset to write the story of the product as well as understanding the sentiments of the investors. Only when the story of the structured product matches the sentiments of the investors will there be business or transactions.

Most of the reference books on structured products either focus too much on technical issues, such as pricing models, or just provide an introduction about the nature of the products. It is difficult to find a book that provides a balanced discussion on technical issues as well as business or managerial issues. This is the motivation for our writing this book: our objective was to write a book that allows readers to understand a wider spectrum of issues faced by practitioners in the domain of structured products.

Topics covered in this book include the following:

- movements of various assets, trends, volatility, correlation, and behavior of assets under extreme market conditions;

- estimation of volatility, skewness, kurtosis, correlation, mean reversion, and the formation of market views;
- volatility smile, volatility surface, and local volatility models;
- volatility index and variance swap;
- mathematical or numerical methods for product pricing;
- survey of equity and foreign exchange structured products.

The book is designed for practitioners as a reference or as a textbook for students at the master's level who feel that plain-vanilla options or the Black—Scholes model based on the normal distribution is no longer adequate and that they need to learn more sophisticated pricing models, for example, stochastic volatility models such as Heston and local volatility models such as Dupire. This book could be used for a short semester course of 7 or 8 weeks if students have already finished one course in derivatives or a long semester of 12 or 13 weeks, so that a few lectures can be used to cover fundamental topics such as options and their pricing. Several research assistants provided significant support in finishing this book. Without their support, this book would not have been completed. They are: Yuen Kwan (Claire) Chan, Ying Lun (John) Cheung, Cheuk Hang (William) Leung, Xinlu Tang, Boyang Zhao, Yilin Yang, and Pinzhi (Michael) Zhou.

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1 Survey and classification of structured products

1.1 Background

Structured products are designed as part of prepackaged investment strategies by financial engineers, often combining elementary financial instruments such as bonds, stocks, futures, and options that are traded independently in spot and futures markets, to offer tailor-made risk return profiles to the investors. These products are designed to help investors pursue a broad range of portfolio and risk management objectives, such as protection, optimization, enhancement of returns, and leverage. They can help diversify portfolios, address various market conditions, or manage risks and taxes.

There are various ways in which structured products can be classified, and we are going to discuss them in detail.

1.2 Literature review

1.2.1 History and product development

In the early 1990s, many investment banks thought up new solutions to attract more investors to equity markets. The idea was to create innovative options (products?) with sophisticated payoffs that would he based on all types of assets such as stocks indices, commodities, foreign exchange, and all kinds of funds. Also, banks were looking for intelligent ways to provide investors with easy access to these innovations by issuing wrappers (medium-term notes, insurance life contracts, and collective funds) in a tax-efficient manner. Moreover, it was important to structure a business that was capable of following an issued financial asset throughout its life. Therefore, structured roles were created to compose complex over the counter products, while quantitative analysts developed pricing models to enable traders to hedge the products until maturity. Banks were also conscious of the importance of providing secondary markets that introduced the liquidity the business needed to expand. (See *Exotic Options and Hybrids: A Guide to Structuring, Pricing and Trading* by Mohamed Bouzoubaa and Adel Osseiran.)

Another school of thought considers that structured investment products came into being because companies wanted to issue debt, implying fewer opportunity costs to the investors. Traditionally, one of the ways to do this was to issue a convertible bond, i.e., debt that under certain circumstances can be converted into equity. In exchange for the potential for a higher return (if the equity value would increase and the bond could be converted at a profit), investors were willing to accept lower interest rates. However this trade-off and its actual worth are debatable because the movement of the equity value of the company was unpredictable. Investment banks then decided to add features to the basic convertible bond, such as increased income in exchange for limits on convertibility of the stock, or principal protection. These extra features were all based on strategies investors themselves could formulate, using options and other derivatives, but these were prepackaged as one product. The goal was again to give investors more reasons to accept a lower interest rate on debt in exchange for certain features. On the other hand, the goal for the investment banks was to increase profit margins because the newer products with added features were harder to value, making it harder for banks' clients to see how much profit the banks were making from it.

1.3 History and market development

Structured products gained popularity in the United States during the 1980s and were introduced in Europe in the mid-1990s, during years of low interest rates. Though the trend came into being because companies wanted to issue debt more cheaply, first, convertible bonds were issued that under certain circumstances could be converted into equity. In exchange for potential higher returns, investors were ready to accept lower interest rates. Investment banks and financial engineers kept innovating different financial instruments to provide more and more reasons for investors to accept lower interest rates, and features such as increased income in exchange for limits on convertibility of the stock or principal protection were added.

The past two decades have seen a huge growth in structured products globally. The 2000–2003 large drops in stock markets were one of the reasons that motivated investors to look at structured product as an influential alternative investment strategy; the other reason was increased market volatility.

When structured products were first introduced to high net-worth individuals in the early 1990s, the value proposition was focused on innovation, access to capital markets, and potential for enhanced performance. Since then, the structured products market has undergone major changes. The number of private investors using them has increased significantly, and a wider range of standardized products has been introduced.

According to the definition given by Roberto Knop, "a structured product is a financial instrument, and its return depends on the composition of other, simpler products. It consists of a loan, and one or more derivative products. The special feature here will be the conversion of the original risks of each of its components."

Sajit Das insists that structured products represent a special class of fixed-income instruments, and their principal appeal is the capacity to generate highly customized exposures for investors, consistent with their investment objectives with predetermined risk return parameters.

Many other similar definitions of structured financial products have been put forth that in turn focus special attention on the fact that an instrument's basis is the combination of a security (or a monetary asset) and one or several derivatives.

However we cannot fully claim that a structured product is a "security packaged together with derivative(s)." The concept of a structured product is so wide that all components of the instrument are not always clear. The usage of derivatives is not necessary. Thus, the Finnish Association of Structured Products asserts that "banks do not include a derivative component and banks are not necessarily using derivatives to hedge the underlying risk, even though there are features in structured products that resemble derivative like behavior."

The following definition of a structured product seems to be the most appropriate for understanding its essence: a structured product is a complex financial instrument with predetermined conditions of payoff and initial capital return, linked to a certain underlying asset. The product payoff depends on the underlying asset's dynamics, and the type of payoff and periodicity are defined by components of the structured product, i.e., securities (mainly fixed income instruments) and derivative instruments.

1.4 The goals and purposes of structured products

The uses of structured products are very broad, including the following:

- **Arbitrage:** Both investors and issuers can carry out arbitrage trades with derivatives and underlying assets by means of structured products.
- **Investment restrictions:** Such groups of investors as pension and mutual funds and insurance companies can access derivatives transactions via structured products.
- **Taxation and accounting:** Structured products are simple from the perspective of accounting and taxation as they are considered as separate security, and the value of derivatives is already included in the product price.
- **Creation of products "a la carte":** The freedom to create products is almost unbounded. Products are customized to fit the unique requirements of investors.
- **Hedging:** Structured products can be used not only for investments but also to hedge positions against market risks.
- **Access to new markets:** With the help of structured products, investors can access exotic instruments and new markets, for example, assets and instruments of developing markets that would otherwise be difficult for investors to access directly.

- **Cheap funding source:** A part of the funds intended for fixed income investment can be used by the issuer for its own financing at rates cheaper than the market rates.

1.5 The classification of structured products

The goals and purposes of structured products define their classification. In modern practice, there is no uniform and universally accepted system of product classification. There are a number of reasons for this: newness of the market, access restrictions among private investors, constant creation of new instruments, and proper interpretation of products by investment banks.

The lack of a standardized classification of structured products has tough implications for all market stakeholders. The consequences are as follows:

1 Difficulties due to the lack of definition from the legal perspective
2 Issue and information disclosure problems
3 Complex risk and yield calculations
4 The placement of restrictions and limits on transactions with products
5 Difficult product distribution
6 Low liquidity.

Of course, in order to solve the problem of classification, general coordination across the entire market of structured products is required. Only then can clear and uniform categories that define the types and classes of assets be determined.

After the products currently available in the market were studied, the following classification was prepared (see Appendix A).

1.5.1 Classification by levels of principal protection

On the basis of degree of protection of the capital, structured products can be divided into the following categories:

- **Principal protected products:** Those that provide full protection of the initial capital, independent on the underlying asset's price movements
- **Partially protected products:** Those that guarantee the return of the initial capital only at a certain level in the form of a percentage of the originally invested sum.

1.5.2 Classification by quantity of payments

- **Coupon products:** Products that provide more than one payment, similar to usual bonds, throughout the lifespan
- **Non-coupon products:** Products that offer only one payment at maturity, which includes both the return of the initial capital and the profit and loss amount.

1.5.3 Classification by type of underlying asset

The following underlying assets can be linked to the product:

- Security
- Interest rate
- Currency
- Index
- Basket of assets (currencies, securities, commodities, etc.)
- Commodities
- Credit quality
- Volatility
- Spread
- The consumer price index and other macroeconomic indicators
- Property price index.

1.5.4 Classification by form of structured product

Structured products can be issued in the following forms:

- Security
- Deposit
- Fund
- Private banking service.

1.5.5 Classification by type of investor

Each structured product is designed for a predetermined group of investors and customers. It is possible to outline three basic groups of investors:

- **Retail group:** Mass consumers
- **Group of institutional investors:** Large investment banks, mutual and pension funds, state funds, etc.
- **Individual investors:** Wealthy consumers.

1.5.6 Classification by behavior of underlying assets

Structured products' payoffs depend on the dynamics of the underlying assets to which they are linked. The behavior models can be defined as follows:

- Growth/falling
- Lateral movement
- Occurrence/non-occurrence of an event
- High/low volatility.

1.5.7 Classification by degree to which payoff depends on price path of underlying asset

Payoffs of structured products can be either defined by the value of a variable at the maturity date or by the value of a variable through the lifespan of a product. Thus, the payoff can be *independent* of and/or *dependent* upon the price path of the underlying asset.

1.5.8 Classification by payoff functions

As noted earlier, the basic peculiarity of structured products is the core element: derivative financial instruments. Almost all derivatives can be used for the creation of structured products. The type of derivatives and their combinations define the payoffs' functions that differentiate one product from another. Having investigated the products offered on the market, the following types of payoff functions can be segregated:

- **Tracking functions:** Payoffs are fully defined by the movement of the underlying asset and a change of 1% provides 1 percent change in the price of the product. Product example: protected tracker.
- **Leveraged functions:** Financial leverage is used. These products bear the risk of a partial loss of the initial capital. Product example: leverage long with sop loss note.
- **Basket functions:** Payoffs are defined by the dynamics of one asset versus a basket of underlying assets. Product example: Altiplano note.
- **Barrier functions:** Payoffs based on reaching or not reaching the underlying asset of a certain barrier level. Product example: knock-in, knock-out note.
- **Functions with floating parameters:** The main parameters of options can be changed (for example, a strike) when the underlying asset has overcome a certain level. Product example: Cliquet note.
- **Fixed payoff functions:** Payments in this case are fixed. Product example: reverse convertible.
- **Swap functions:** Within those functions, the payoffs are defined by spreads between prices (values) of certain underlying assets or by their volatility. Product example: dispersion note.

The disclosure of mentioned indicators and their detailed descriptions will allow all market participants to outline more accurately the limits and possibilities of the market's functioning and further development.

1.6 Case analysis

We end this chapter by giving some examples of how we can classify a product using the foregoing criteria. The examples chosen are real financial products. Before introducing the classification, let us introduce the mechanism of the products first.

1.6.1 Non-deliverable swap

A non-deliverable swap is also called a single-/double-touch quanto swap. This product has the following characteristics:

- **Currency:** There are two types of currency, domestic currency and foreign currency, and they are agreed to by both counterparties.
- **Underlying asset:** In our example, the underlying asset is an FX rate. Such an FX rate may not be the domestic currency and the foreign currency agreed at the beginning.
- **Payment of the swap:** There are two payment legs of the swap: *funding leg* and *coupon leg*. The buyer of the swap pays a contractual agreed fixed rate to the seller of the swap. On the other hand, the seller of the swap pays a contractual agreed coupon rate to the buyer of the swap. Hence, the buyer of the swap is said to pay a funding leg and receive a coupon leg, while the seller of the swap is said to pay a coupon leg and receive a funding leg.
- **Barriers:** There are two pairs of barrier levels. These barriers are used as criteria for determining the amount of coupon payments received by the buyer of the swap.
- **FX rate:** Rate at which the underlying currency is converted to foreign currency.

At the fixing date, the buyer of the contract pays a fixed amount on receiving different coupon payments according to different pre-agreed situations that are settled in foreign currency.

1.7 Mechanism of non-deliverable swap

In this section, two situations are investigated: double touch and single touch.

1.7.1 Case 1 (double touch)

To be specific, the following notations are defined:

1. Two pairs of barriers: UB1 and LB1 (pair 1 barrier) and UB2 and LB2 (pair 2 barrier).
2. Three different coupon rates: R_{C1}, R_{C2}, and R_{C3}.
3. Funding rates: R_C.
4. Two foreign exchange rates: FX_1 and FX_2.
5. Notional: N.

The following rules must also be obeyed:

1. FX_1 and FX_2 are the USDCNY and EURUSD FX rates, respectively. However, the roles of FX_1 and FX_2 are different. The FX rate FX_1 is treated as the exchange rate for settlement, whereas the FX rate FX_2 is treated as the underlying asset.

2 Initially, UB1 > UB2 > *Spot* FX_2 > LB2 > LB1.
3 Domestic currency is treated as CNY, and foreign currency is treated as USD.

If $FX_2 > UB1$ and $FX_2 < LB1$ during the index barrier observation period, then the coupon rate will be R_{C1}. However, if $FX_2 < UB1$ and $FX_2 > LB1$, and at the same time it turns out out that $FX_2 > UB2$ or $FX_2 < UB2$, then the coupon rate will be R_{C2}. Otherwise, the coupon rate will be R_{C3}.

The following figures illustrate the mechanism of how the product works. Figure 1.1 illustrates the scenario in which the coupon rate R_{C1} will be received, whereas Figure 1.2 illustrates the scenario in which the coupon rate R_{C3} will be employed. Figure 1.3 gives one interpretation of when R_{C2} will be given.

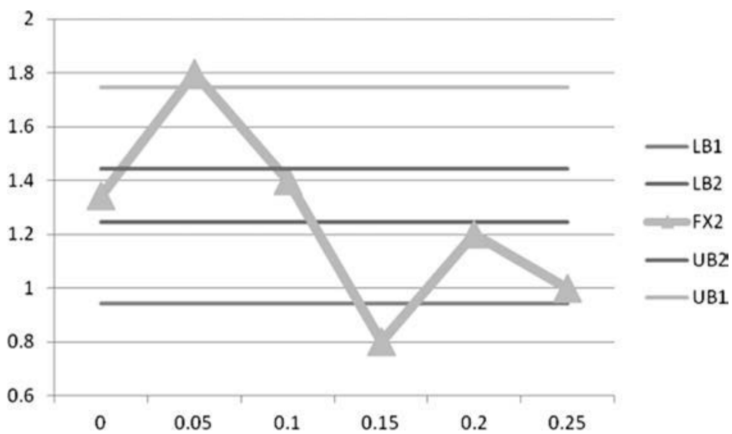


Figure 1.1 Situation in which coupon rate R_{C1} is given.

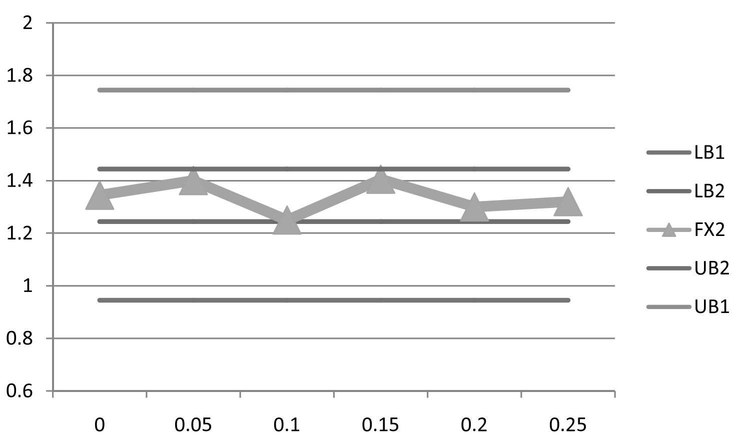


Figure 1.2 Situation in which coupon rate R_{C3} is given.

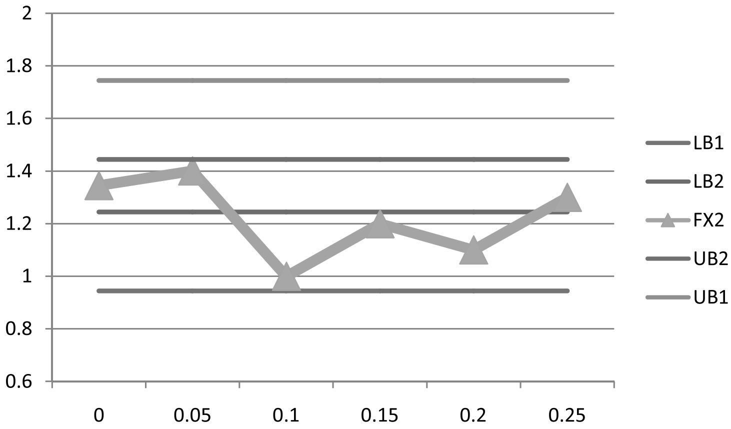


Figure 1.3 One interpretation in which coupon rate R_{C2} is given.

1.7.2 Case 2 (single touch)

All notations and conditions defined in Case 1 can be used for this case also. However, coupon payments are completely different.

If $FX_2 > UB1$ or $FX_2 < LB1$ during the index barrier observation period, then the coupon rate will be R_{C1}. However, if $FX_2 < UB1$ and $FX_2 > LB1$, and at the same time $FX_2 > UB2$ or $FX_2 < UB2$, then the coupon rate will be R_{C2}. Otherwise, the coupon rate will be R_{C3}.

The following figures illustrate the mechanism of how the product works. Figures 1.4 and 1.5 give two independent interpretations in which the coupon rate R_{C1} is given. Figure 1.6 gives an interpretation of when R_{C2} will be given, whereas Figure 1.7 illustrates the scenario in which the coupon rate R_{C3} will be employed.

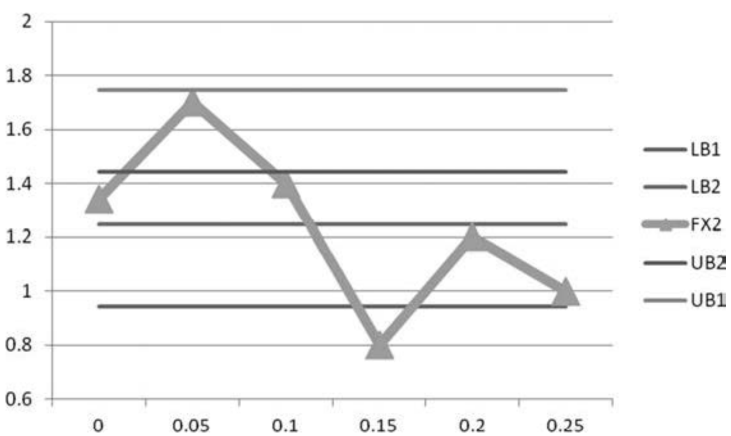


Figure 1.4 An interpretation in which R_{C1} is given.

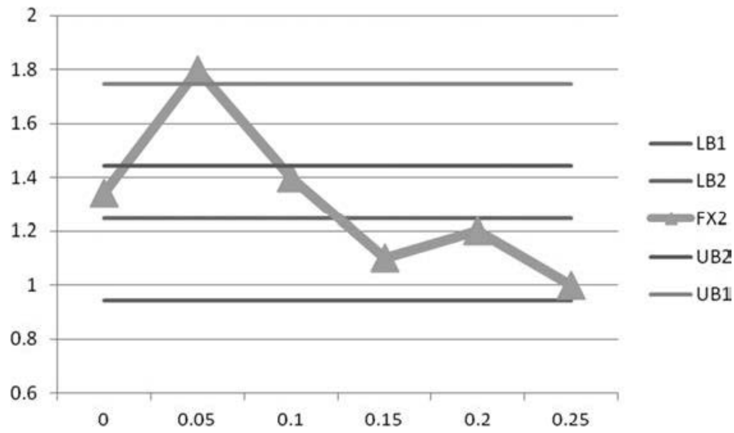


Figure 1.5 Another interpretation in which R_{C1} is given.

1.8 Basic analysis for buyer

The product mentioned promises a positive and CNY-based cash flow to the investor. It also offers a tool to bet on the volatility of the underlying FX. To be specific, assuming that $R_{C1} > R_{C2} > R_{C3}$, the buyer of the contract bets for high volatility because the probability of triggering the "pair 1 barrier" becomes larger if the volatility is high, which results in getting the high coupon rate R_{C1}. Hence, the payoff of the buyers becomes larger if the volatility of the underlying asset is high. On the other hand, if R_{C3} is the largest, the buyer of the contract bets for low volatility because the probability of not triggering both "pair 1 barrier" and "pair 2 barrier" becomes smaller if the volatility is high. Thus, the payoff of the buyers becomes larger if the volatility of the underlying asset is low.

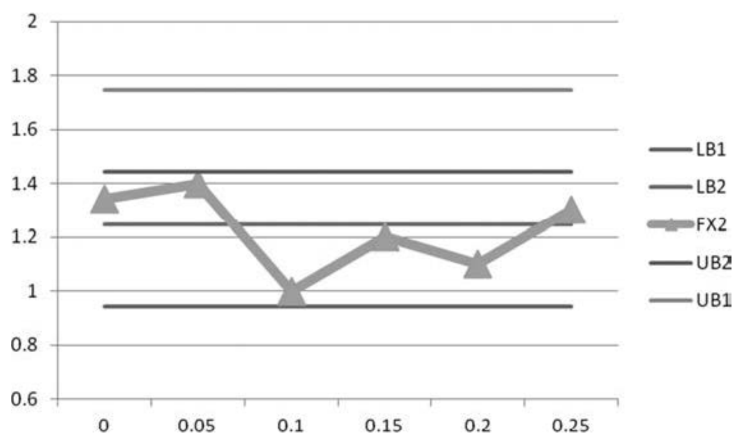


Figure 1.6 Another interpretation in which R_{C1} is given.

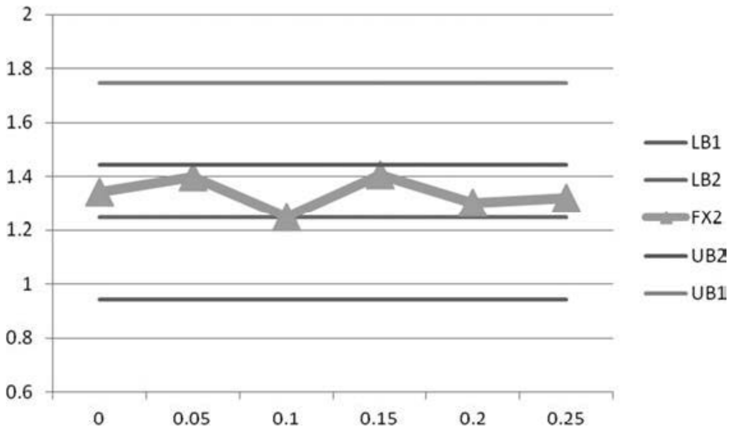


Figure 1.7 The scenario in which R_{C3} is given.

Although there are two cases of the non-deliverable swap, both lead to the same conclusion under the aforementioned classification criteria, which can be summarized in Table 1.1.

1.9 Auto-callable ratio par forward

The auto-callable ratio par forward is associated with the underlying asset, the FX rate. Such a contract is a synthetic forward consisting of a call and put that have the same strike and expiry date but different notionals.

In addition, the product has three different variations:

1 Multiple knock-out type (or American knock-out type)
2 Guarantee type (based on American knock-out type)
3 Bonus type.

Table 1.1

	Non-deliverable swap
By levels of principal protection	Partially protected product
By quantity (periodicity) of payments	Coupon product
By the type of underlying asset	FX rate
By the form of a structured product	Private banking service
By the type of investor	Individual investor/group of institutional investor
By behavior of underlying asset	Occurrence/non-occurrence of triggered event
By the degree to which the payoff depends on the price path of the underlying asset	Independent of the price path
By the payoff functions	Barrier function

1.9.1 Mechanism of auto-callable ratio par forward

To be specific, it is assumed that there are 22 fixing dates, Party A is the seller, and Party B is the investor (buyer). The entire period is divided into two periods: the first period includes the first 11 periods, and the second period includes the last 11 periods. Moreover, the following terms are defined:

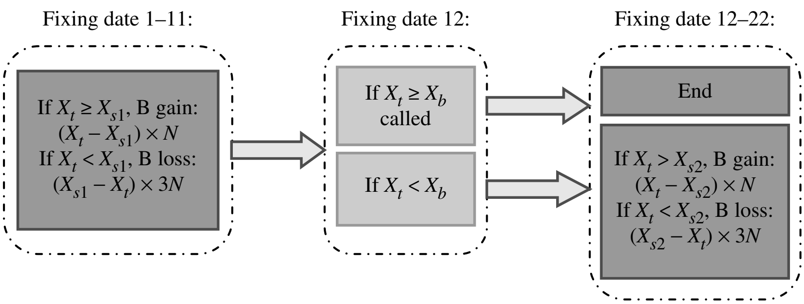
1 X_t is the FX rate.
2 N is the notional in domestic currency.
3 X_{s1} is the strike rate for the first period.
4 X_{s2} is the strike rate for the second period.
5 X_b is the barrier rate.

There are different types of auto-callable ratio par forward, which are now described.

1.9.2 The original (one-time knock-out) type

For this type, the product can be called at the 12th fixing date. On each fixing date in the first period, if the FX rate X_t is greater than or equal to X_{s1}, then the buyer earns $\left(X_t - X_{s1}\right) \times N$. However, if the FX rate X_t is less than X_{s1}, then the buyer loses an amount equal to $\left(X_{s1} - X_t\right) \times 3N$. On the 12th fixing date, if the FX rate is greater than or equal to the barrier rate, the product is called by the seller, and the contract is terminated. On the other hand, if the FX rate is smaller than the barrier rate, the contract can continue, and on each fixing date in the second period, the buyer gains $(X_t - X_{s2}) \times N$ if $X_t > X_{s2}$ and the buyer loses $\left(X_{s2} - X_t\right) \times 3N$ if $X_t < X_{s2}$.

The cash flow on each fixing date can be summarized in the following flow chart:

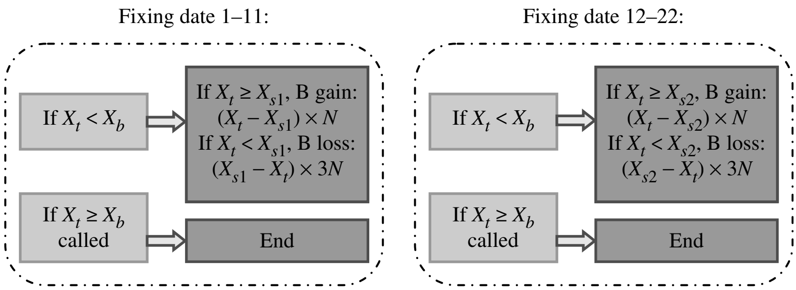


1.9.3 Multiple knock-out (or American knock-out) type

This type can be called by the seller on each fixing date, which is determined by whether the FX rate X_t is greater than or equal to the barrier rate X_b.

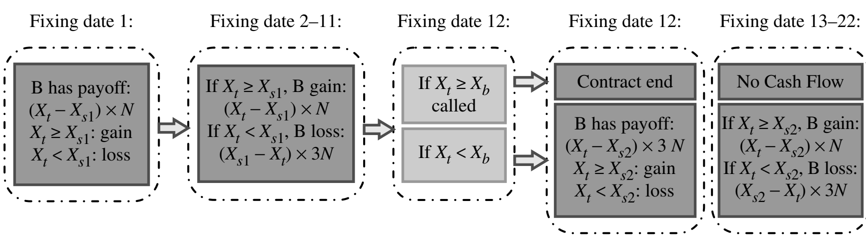
If the product is not called, then in the first period, the buyer can gain $\left(X_t - X_{s1}\right) \times N$ when $X_t \geq X_{s1}$ and loses $\left(X_{s1} - X_t\right) \times 3N$ when $X_t < X_{s1}$. On the other hand, in the second period, the buyer can gain $(X_t - X_{s2}) \times N$ if $X_t \geq X_{s2}$ and loses $\left(X_{s2} - X_t\right) \times 3N$ if $X_t < X_{s2}$.

The cash flow on each fixing date can be summarized in the following flow chart



1.9.4 The guarantee type (based on American knock-out)

Properties and cash flows of this type are similar to the American knock-out (multiple knock-out) type, except that on the first fixing date, an auto-call is not available, and the buyer gains $\left(X_t - X_{s1}\right) \times N - X_t \geq X_{s1}$ and loses $\left(X_t - X_{s1}\right) \times N \_ X_t < X_{s1}$.

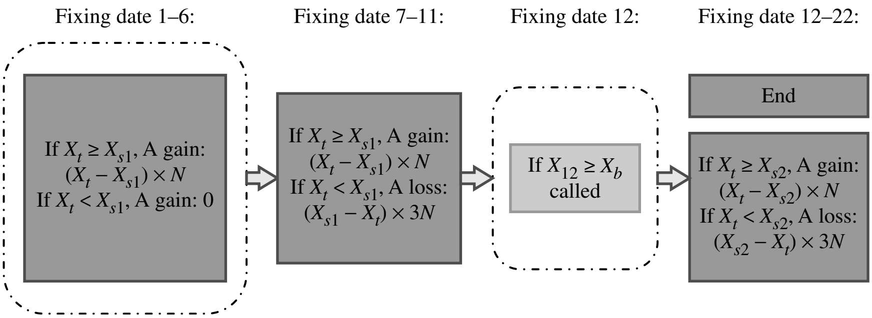


1.9.5 The bonus type

For this type, the callable feature is the same as the original (or one-time knock-out) type. However, cash flows are completely different on each fixing date. Instead of considering two periods, there are three periods: 1st period from fixing day 1 to fixing day 6, 2nd period from fixing day 7 to fixing day 11, and the last period from fixing day 12 to fixing day 22.

In the first period, the buyer gains $\left(X_t - X_{s1}\right) \times N$ if $X_t \geq X_{s1}$ and gets nothing if $X_t < X_{s1}$. In the second period, the buyer gains $\left(X_t - X_{s1}\right) \times N$ if $X_t \geq X_{s1}$ and loses $\left(X_{s1} - X_t\right) \times 3N \;-\; X_t < X_{s1}$. In the last period, the buyer gains $(X_t - X_{s2}) \times N$ _ $X_t \geq X_{s2}$ and loses an amount equal to $\left(X_{s2} - X_t\right) \times 3N - X_t < X_{s2}$.

The cash flow on each fixing date can be summarized as follows:



1.9.6 Basic analysis of one-time knock-out type

Because of the difference in the notional, the auto-callable ratio par forward indeed has a larger leverage of loss rather than gain. So the investor should have a strong market view that the FX rate will stay in the benefit range (above strike if the investor calls foreign currency, below strike if the investor puts foreign currency) during the target period. Otherwise, the investor will lose more money. And if the FX rate goes out of the benefit range at the first fixing date of the second period (like the 12th fixing date in our sample), it will not be called after the first fixing date of the second period, and what is worse, it will continue to lose money until the last fixing date of the second period. The auto-callable barrier is beneficial to the seller as it guarantees that the seller of the RPF will not lose too much. Once the FX rate is deep in the money at the callable date (like over the barrier in our sample), the product will be knocked out. So the features of leverage and auto-call are both beneficial to the seller only.

If the FX rate stays in the benefit range in the target period, which means the FX rate behaves well and does not fluctuate much, the buyer will enjoy a good return.

Again, we focus on the classification of the product. With the classification criteria, we obtain Table 1.2.

Table 1.2

	Auto-callable ratio par forward
By levels of principal protection	Partially protected product
By quantity (periodicity) of payments	Coupon product
By the type of the underlying asset	FX rate
By the form of the structured product	Private banking service
By the type of investor	Individual investor
By the behavior of the underlying asset	Occurrence/non-occurrence of triggered event
By the degree that the payoff depends on the price path of the underlying asset	Independent of the price path
By the payoff functions	Barrier function

Appendix A

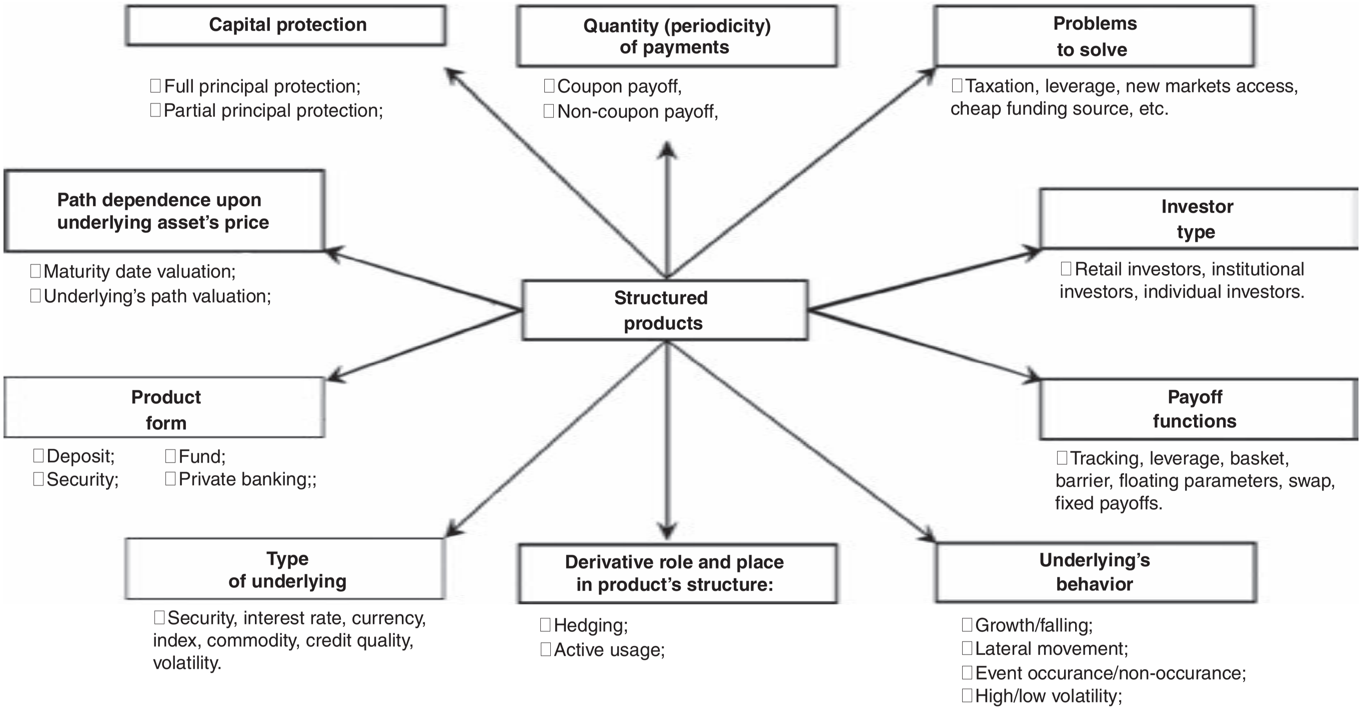


Figure A.1 Classification of structured products.

Further Reading

1 V. Omelchenko. 2009. Definition and classification of structured financial products. http://en.wikipedia.org/wiki/Structured_product#Origin.
The classification of a structured product in economics: http://www.ukessays.com/essays/finance/the-classification-of-a-structured-product-in-economics-finance-essay.php.

2 Tools and methods for pricing exotic options

2.1 Background

Options trading became popular since, 1973, after the Chicago Board Options Exchange (CBOE) standardized and integrated options transactions. In the following 20 years or so, the options market expanded significantly, and many different exotic options were developed. In general, for a particular exotic option, a corresponding closed formula might not exist there, too. Tools/methods need to be explored to price exotic options.

Fischer Black and Myron Scholes are most likely the two economists who first made a breakthrough in derivatives pricing. They developed an analytical model now popularly known as the *Black–Scholes model (BS model)*, which opened the area of research on option pricing.

In this chapter, unless otherwise specified, the following assumptions are made.

2.1.1 Assumptions for BS model

1 Efficient markets (market movements cannot be predicted).
2 Commissions are non-existent.
3 Interest rates do not change over the life of the option (and are known).

To be specific, the following notations are used in this chapter:

S = underlying stock price
X = strike price
r = risk-free interest rate
V = volatility
τ = time to maturity
D = dividend
T = maturity.

Moreover, the stock price is assumed to follow the lognormal distribution:

$$\frac{dS}{S} = \mu dt + \sigma dZ_t^Q,$$

where Z_t^Q is a risk-neutral standard Weiner process, μ is the drift rate, which in the risk-neutral world is given as.

With the above assumptions, it is now possible to explore different tools or models for pricing options. How to select the appropriate models or methodologies? Such selection can be based on the duration of the contract, the purpose of such pricing, and how accurate the pricing reflects the market condition. In addition, if constant volatility needs to be used, then how can the volatility be estimated so that its pricing would not be too far from the one obtained from the market? Normally for products with a short duration, such as 1 week or 10 days, constant volatility can be the best choice, and such volatility can be estimated from the combination of historical volatility and implied volatility that derived from options that traded on the market. For longer duration products, it is too risky or inaccurate to use those models that are based on constant volatility. Any information that can be used to capture the views of the option contract holders, which include their expected asset price and volatility movement, such as volatility term structure, interest term structure, and so on, are welcome to be included in the pricing. Therefore, the volatility is no longer a constant but a stochastic one.

2.2 European option

According to Wikipedia, an option is a contract that gives the buyer (the owner) the right, but not the obligation, to buy or sell an underlying asset at a specified strike price on or before a specified date. The seller has the corresponding obligation to fulfill the transaction, which is to sell or buy the underlying asset, if the buyer decides to "exercise" the option. The buyer pays a *premium* to the seller for this right. An option that conveys to the owner the right to buy an underlying asset is referred to as a call; an option that conveys the right of the owner to *sell* an asset is referred to as a put.

Options pricing is a research area with a long history. In general, the value of an option commonly consists of two parts:

- The first part is the intrinsic value, which is defined as the difference between the market value of the underlying asset and the strike price of the given option.
- The second part is the time value, which depends on a set of other factors that, through a multi-variable, non-linear interrelationship, reflect the discounted expected value of that difference at expiration.

The formula for a European call option on a non-dividend stock is

$$c_t = Se^{-\mathrm{D}\tau}N(d_1) - Xe^{-\mathrm{r}\tau}N(d_2),$$

whereas for a European put option on a non-dividend stock the formula is

$$p_t = Xe^{-\mathrm{r}\tau}N(-d_2) - Se^{-\mathrm{D}\tau}N(-d_1),$$

where $N(d_1)$ and $N(d_2)$ are the cumulative normal distribution functions for d_1 and d_2, which are defined as

$$d_1 = \frac{\ln\left(\frac{S_t}{X}\right) + \left(r - D + \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}}, \quad d_2 = \frac{\ln\left(\frac{S_t}{X}\right) + \left(r - D - \frac{1}{2}\sigma^2\right)\tau}{\sigma\sqrt{\tau}};$$

d_2 can be further simplified as

$$d_2 = d_1 - \sigma\sqrt{\tau}.$$

In addition, the volatility of the stock price is an important input, which can be estimated from historical data, which is known as historical volatility; however, it only reflects the past history and consists of no view of the future. A more practical method that utilizes the additional information about the view of the option buyers is to use implied volatility that is extracted from the market traded options, especially those put options based on the Black-Scholes Formula, and use numerical methods like the Newton-Rhapson or simplified methods like approximation (Brenner and Subrahmanyam, 1988).

Having outlined the foundations of option pricing via the BS framework, we can describe numerous other methods to price European options and the underlying theories.

Here we look at some other models to price standard vanilla European options.

2.2.1 Pricing

2.2.1.1 Binomial method

The binomial method is a common method to price all types of options, both vanilla and the more exotic types owing to its flexibility (Cox, Ross, Rubinstein, 1979). European options can be related to the BS model, for reasons discussed later.

First, we consider the fundamental properties of the binomial method. It essentially models the movement of the stock price over time and hence the option price by considering the movements at each time node where the up (u) and down (d) "jumps" are given as

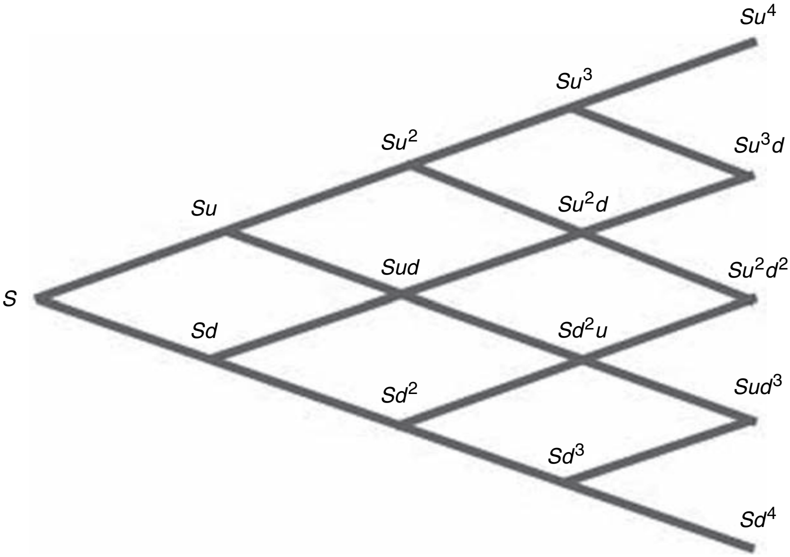
$$u = e^{\sigma\sqrt{\tau}} \quad \text{and} \quad d = e^{-\sigma\sqrt{\tau}}.$$

From the formula u and d, it can be seen that ud=1. The probability of an upward stock movement (i.e., increasing) is

$$p = \frac{e^{(r-D)\tau} - d}{u - d}.$$

For the probability of a downward stock movement, note that the combined probabilities must equal 1; hence, the probability of the downward movement is $1 - p$.

An illustration of how the binomial tree works is shown in the following example:



Having found the value of stock price at the end of the nodes, we can calculate the option value by means of backward induction, that is, working from the far right of the lattice, back to the origin. A simple computer algorithm is able to solve the option values with a large number of nodes. For European call and put options, the binomial model is given as

$$c = e^{\sigma\sqrt{\tau}} \sum_{i=j}^{n} \left(\frac{n!}{i!(n-i)!} \right) p^{\mathrm{i}} (1-p)^{\mathrm{n-i}} \left(Su^{\mathrm{i}} d^{\mathrm{n-i}} - X \right)$$

and

$$p = e^{\sigma\sqrt{\tau}} \sum_{i=0}^{\mathrm{j}-1} \left(\frac{n!}{i!(n-i)!} \right) p^{\mathrm{i}} (1-p)^{\mathrm{n-i}} \left(X - Su^{\mathrm{i}} d^{\mathrm{n-i}} \right),$$

where j is given by the following condition:

$$j = \Phi \left(\frac{\ln\left(X / Sd^{\mathrm{n}} \right)}{\ln(u/d)} \right).$$

Here ϕ represents the next non-negative integer greater than the term in brackets.

There is a correlation between the binomial model and the BS pricing model in the context of the valuation of options. With a significant number of time nodes, the binomial method begins to converge, and the convergence of this value ultimately becomes the value obtained from the closed-form formula.

2.2.1.2 Trinomial method (Boyle)

The trinomial method of pricing options was introduced by Boyle (1986); it is an attempt to model stock price movements better than the binomial method. As one can guess by its name, the trinomial method is similar to the binomial lattice in that the stock price is modeled by a tree, but instead of two possible paths, the trinomial tree has three paths: up, down, and a stable path. The probabilities of each are

$$p_d = \left(\frac{e^{\sigma\sqrt{\tau/2}} - e^{(r-D)\tau/2}}{e^{\sigma\sqrt{\tau/2}} - e^{-\sigma\sqrt{\tau/2}}} \right)^2, \quad p_u = \left(\frac{e^{(r-D)\tau/2} - e^{-\sigma\sqrt{\tau/2}}}{e^{\sigma\sqrt{\tau/2}} - e^{-\sigma\sqrt{\tau/2}}} \right)^2, \quad p_m = 1 - p_d - p_u.$$

The probabilities can also be represented as

$$p_d = -\sqrt{\frac{t}{12\sigma^2}}\left(r - \frac{1}{2}\sigma^2\right) + \frac{1}{6}, \quad p_u = \sqrt{\frac{t}{12\sigma^2}}\left(r - \frac{1}{2}\sigma^2\right) + \frac{1}{6} \quad \text{and} \quad p_d = \frac{2}{3}.$$

Note also that the trinomial method converges much faster than the binomial method.

2.2.1.3 Adaptive mesh model

Normally, constant volatility is used in such pricing with trees. However, for pricing options with a longer duration, information of volatility term structure is needed to make the pricing closer to the market, or to reflect the views of the investors that reflected in the prices (volatility) that they were willing to offer for options with different durations. The Boot-trap approach can be used to extract the volatility to be used in a different time-interval. If such volatility fluctuated significantly, then constant mesh size would not provide sufficient accuracy, and an adaptive mesh model can be a good solution for such a situation. Also, for pricing exotic options like a barrier option, when the asset size is getting close to the barrier, the price changes very significantly. Therefore, using the smaller mesh size can more accurately capture the price change in such an area.

Adaptive Mesh Model (AMM) is a very flexible approach that helps in enhancing the efficiency in trinomial trees to a great extent. It uses coarse time and price steps in most of the tree when market or data points are smooth, but small sections of finer mesh are used in more volatile regions to improve resolution in such critical areas. In other words, the model is based on a lattice-like numerical process in which regions of high resolution are embedded in critical regions of a lower resolution mesh.

The AMM is a variant of the standard lattice models in that the mesh size can be adjusted based on the market condition; for example, the volatility of stock prices.

Compared with the standard lattice approach, AMM provides a faster convergence and more accurate results. However, an indicator needs to be determined and calculated to guide the adjustment of the mesh size. Although the adaptive mesh is more common in pricing path-dependent options such as barrier options,