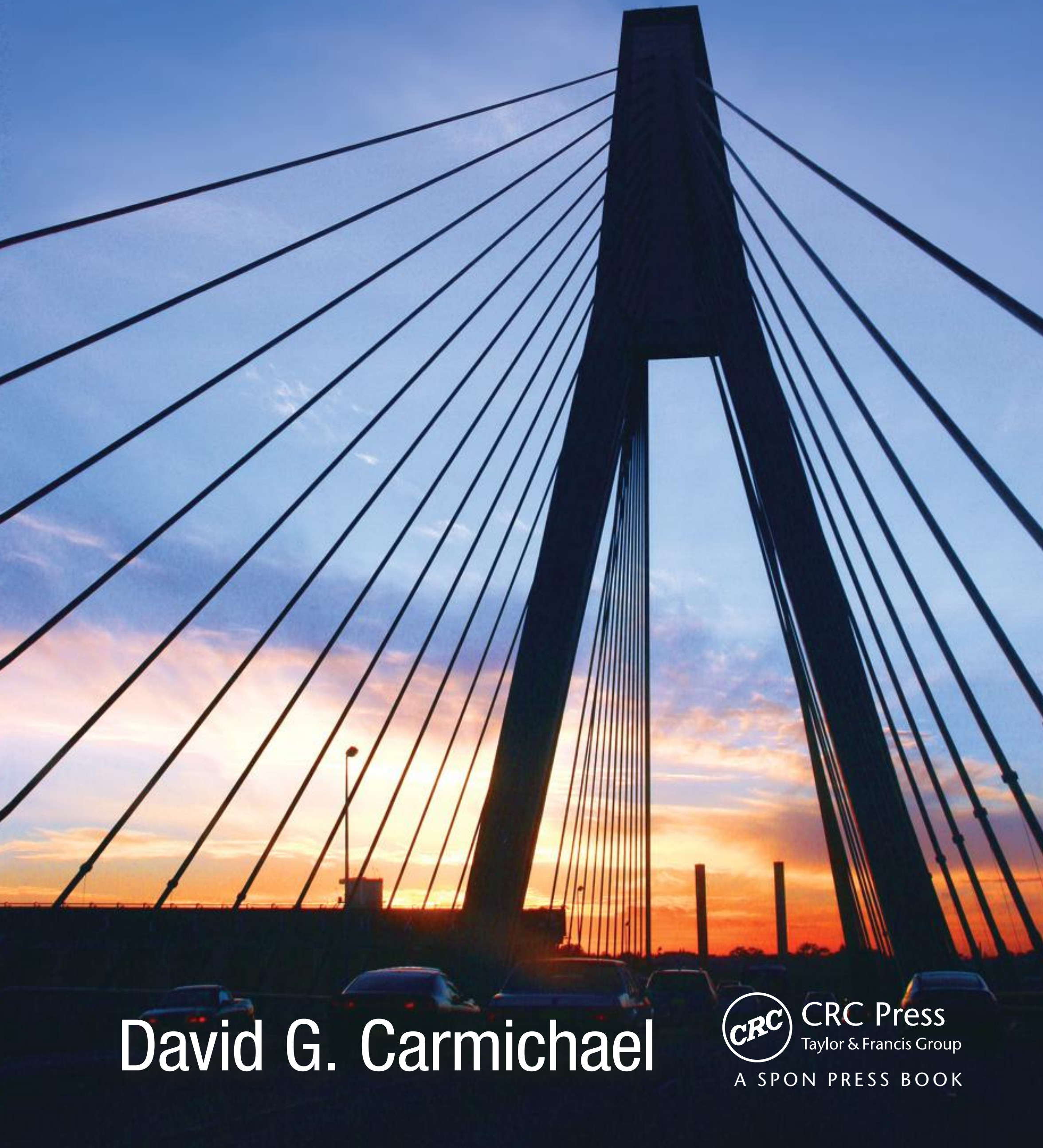
Infrastructure Investment

An Engineering Perspective



David G. Carmichael

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Contents

Preface

For any potential infrastructure or asset, such as investments, it is necessary to establish viability, both in monetary and nonmonetary terms. *Infrastructure* and *assets* here refer to buildings, roads, bridges, dams, pipelines, railways and similar, and facilities, plants, equipment and similar. Nonmonetary issues have, for example, social, environmental and technical origins.

Future demand and future costs for infrastructure and assets generally are uncertain in both timing and magnitude. As well, other investment viability analysis input such as interest rates are similarly uncertain. The book provides the methodology by which the viability of infrastructure or any asset may be appraised as investments, given future uncertainty.

In comparison, most conventional and existing practices ignore uncertainty or attempt to include it in deterministic (that is, assuming certainty) ways, often nonrationally. For example, using interest rates adjusted for investment uncertainty is common practice, as is sensitivity analysis. Business-as-usual might be assumed to repeat in the future, even though everyone knows this is not true, but it is convenient for people to assume so, and because usable approaches incorporating true uncertainty have heretofore been unavailable. Even climate change is commonly treated deterministically in existing approaches.

The book has particular applicability for decision makers presently struggling with analyzing investments with uncertain futures, including the impact of climate change and the possible use of adaptive and flexible solutions, capable of responding to changed futures, and how such uncertainty impacts the future performance of these investments.

The book represents an original contribution to investment viability analysis under uncertainty. Existing texts have not ventured into this territory, but have rather stayed with restrictive deterministic treatments. Additionally, the book gives a very systematic and ordered treatment of its subject matter. The level of required mathematics is no more than that which is familiar to undergraduates.

The formulations given provide interesting insight into investment viability calculations and will be of use to practitioners engaged in investigatory work, especially those looking at investment risk. The material presented on options analysis opens this area to all users, breaking the confines of existing financial options analogies.

The book will be of interest to students, academics and practitioners dealing with decision making on infrastructure, assets and like investments. It will be of interest to those engaged in investments, and the analysis of real options and financial options. The content is presented in straightforward terms in order to ensure as wide a readership as possible.

The book leads the reader through a structured flow, from a systematic treatment of conventional deterministic approaches to a complete probabilistic treatment incorporating uncertainty.

About the author

David G. Carmichael is professor of civil engineering and former head of the Department of Engineering Construction and Management at the University of New South Wales, Australia. He is a graduate of the Universities of Sydney and Canterbury; a fellow of the Institution of Engineers, Australia; a member of the American Society of Civil Engineers; and a former graded arbitrator and mediator. Professor Carmichael publishes, teaches and consults widely in most aspects of project management, construction management, systems engineering and problem solving. He is known for his left-field thinking on project and risk management (*Project Management Framework*, A. A. Balkema, Rotterdam, 2004), project planning (*Project Planning, and Control*, Taylor & Francis, London, 2006), and problem solving (*Problem Solving for Engineers*, CRC Press/Taylor & Francis, London, 2013).

Chapter 1

Introduction

1.1 APPRAISAL

The appraisal of potential infrastructure or assets, as investments, looks at the *benefits* and *costs* of everything related to the investments, both now and into the future. Common benefits include ongoing rental or product sales, and salvage or residual value at end-of-life. Common costs include initial capital cost; ongoing operation and maintenance costs; refurbishment, renovation or retrofitting costs; and disposal costs at end-of-life. *Infrastructure* and *assets* here refer to buildings, roads, bridges, dams, pipelines, railways and similar, and to facilities, plant, equipment and similar. An appraisal of a potential investment assists in

- (For a *single investment*, or for each investment) Establishing whether it is worthwhile proceeding with the investment. In other words, is the investment *viable*?
- (For *multiple investments*) Selecting between alternative investments (*preference*). In other words, which is the best investment?

Other names for *appraisal* include evaluation, study, analysis, feasibility study, benefit–cost analysis, and cost–benefit analysis. Of the last two names, engineers tend to use the second last version and stress the benefits side of the equation, much as engineers prefer to look at a glass half full, rather than half empty.

An appraisal involves consideration of issues that are

- Financial
- Nonfinancial (for example, environmental, social and technical)

That is, benefits and costs can be wider than just money. The social and environmental issues might be called *intangibles* and will have units of measurement that are not money. The units of measurement of benefits and costs need not be money, though many people find it hard or impossible thinking of units of measurement other than money. This rigidity in thinking is changing with time as environmental and social issues become more dominant in the eye of the public and start to enter people's thinking.

An appraisal can be carried out for every *stakeholder* involved in a potential investment. The data feeding into each stakeholder appraisal, of course, will be different depending on the concerns of the stakeholder. That is, it is possible for each stakeholder to come to a different conclusion as to viability and to the preferred investment. This raises serious issues of how the different viewpoints of different stakeholders are resolved, particularly in any investment that impacts on the public and on public interest (pressure) groups.

A financial appraisal would generally only involve items that can be expressed directly in money units and that affect the balance sheet and cash flow (money coming in and money going out) of the stakeholder. Typically, financial appraisals are done by the private sector. An economic appraisal, on the other hand, involves intangibles comprising environmental and social concerns, technical performance and so on, as well as the direct money items. An attempt may be made to put a monetary value on social and environmental items, but this is controversial. Typically, economic appraisals are done by the public sector. In some circumstances, a financial appraisal might be regarded as a special case of an economic appraisal. But there are multiple uses of the terminology. The mathematical manipulations for the financial and economic appraisals are the same if the intangibles are converted to money equivalents. Commonly, all benefits and costs are converted to a money unit for convenience, though the rationale behind this conversion is questioned by many people. The material in this book refers to both of what might generally be called economic appraisals and financial appraisals.

1.2 OUTLINE

This book gives the tools necessary for the appraisal of investments in infrastructure and other assets. The emphasis in this book is on dealing with uncertainty in investments (Part II: Probabilistic). However, it is considered that an introduction to deterministic analysis (Part I: Deterministic) is useful for understanding purposes. Most people learn by going from the particular to the general, and hence understanding Part I represents a necessary step in order to understand Part II.

'The term "probabilistic" implies some variability or uncertainty. "Deterministic", on the other hand, implies certainty. Deterministic variables are commonly described in terms of their mean, average or expected values. Probabilistic variables are commonly described in terms of probability distributions, or if using a second order moment approach, in terms of expected values and variances (standard deviation squared); the variances capture the variability information or uncertainty. Risk, for example, only exists in the presence of uncertainty, and hence risk approaches are probabilistic. With certainty, there is no risk. Generally, determinism is simpler to deal with and, wherever possible, people simplify from probabilistic to deterministic approaches' (Carmichael, 2013).

1.2.1 Part I: Deterministic

Part I is a stepping stone to Part II. It introduces many of the necessary terms used in investment appraisal. It discusses interest (and discount) rate matters, including compound interest for single and series amounts. A brief review of the concepts of compounding and discounting, and discounting equations is given. This leads to the various measures of appraisal, all of which rely on the time value of money. Information on investment viability and preference and the use of discount or compound interest equations in the various measures of appraisal is given. Last, Part I looks at issues to do with the choice of interest (and discount) rates, inflation, nonfinancial matters, sensitivity and benefits. A discussion of how investors treat uncertainty within a deterministic framework is given. Broadly, all these topics might be identified as belonging to what is called discounted cash flow (DCF) analysis.

1.2.2 Part II: Probabilistic

Part II extends the deterministic investment analysis of Part I to explicitly include uncertainty, and to include it in a proper way, rather than by adjusting deterministic thinking in an ad hoc way. This implies a probabilistic formulation as the most rational way forward. The material covers analysis involving probabilistic cash flows, interest rates and investment lifespans, and shows how this can also be used in the valuation of options. The favoured and adopted way of incorporating uncertainty into the analysis in this book is through a second order moment approach where variables are characterized solely in terms of their expected values and variances, thereby not requiring information on probability distributions, which in most applications would not be available.

1.2.3 Common formulation

The common probabilistic formulation given in this book covers many applications, with each application naturally specializing it in different ways. Any investment is converted to a collection of cash flows. A spreadsheet is all that is needed to perform the calculations.

Consider a general investment, with possible cash flows extending over the life, n, of the investment. Let the net cash flow, X_i, at each time period, i = 0, 1, 2, ..., n, be the result of a number, k = 1, 2, ..., m, of cash flow components, Y_{ik}, which can be both revenue and cost related. That is,

$$X_i = Y_{i1} + Y_{i2} + \ldots + Y_{im} \tag{1.1}$$

where Y_{ik}, i = 0, 1, 2, ..., n; k = 1, 2, ..., m, is the cash flow in period i of component k.

Introduce a *measure* called present worth (PW), which is the present-day value of these future cash flows. As shown in Chapter 2, the present worth is the sum of the discounted X_i, i = 0, 1, 2, ..., n, according to,

$$PW = \sum_{i=0}^{n} \frac{X_i}{(1+r)^i} \tag{1.2}$$

where r is the interest rate (expressed as a decimal, for example a rate per period of 5 % is expressed as 0.05). The term 'discounting' (and the related 'discounted') refers to converting future cash flows to their present-day value through the medium of the interest rate, which reflects the time value of money. A measure such as PW may be used to establish investment viability, and for selecting (preference) between alternative investments.

From this, information on PW, other measures such as internal rate of return (IRR), payback period (PBP) and benefit:cost ratio (BCR) can be derived. These other measures may also be used to establish investment viability, and for selecting between alternative investments. All of this is explained in detail throughout the book.

Equations (1.1) and (1.2) can be used for both the deterministic and probabilistic cases. For the probabilistic case, Y_{im}, X_i, r, n, PW, IRR, PBP and BCR become random variables. For the deterministic case, however, this formulation is perhaps more formal than it needs to be. For the deterministic case, many people find it more convenient to think in terms of benefits (B) and costs (C) rather than cash flow components (Y) or cash flows (X).

The obtained information on measures such as present worth and internal rate of return feed into the decision making regarding investments. As such, these measures might be used as objective functions, or interpreted as constraints to be satisfied in the relevant decision-making process (Carmichael, 2013). When viewed as objective functions, what is desired is that investment that maximizes or minimizes, as the case may be, the measures of PW, IRR, PBP and BCR. When viewed as constraints, what is desired is an investment that is less than or greater than, as the case may be, given values of these measures.

1.2.4 Useful reference

Understanding investment analysis is facilitated if a systems or systematic problem-solving basis is used. In this sense, recommended collateral reading is *Problem Solving for Engineers*, D. G. Carmichael, CRC Press, Taylor & Francis, 2013, ISBN 9781466570610, Cat: K16494.

Part I

Deterministic

A systems-style treatment of established deterministic investment appraisal.

Chapter 2

Benefits, costs and time value

2.1 SINGLE AND MULTIPLE INVESTMENTS

When appraising a single potential investment, what the investor is looking to establish is whether the investment is worthwhile, equivalently its *viability*. Alternatively, the terms *feasibility* or *screening* process (separating viable from nonviable investments) might be used.

When comparing multiple potential investments, the investor is looking to establish the best or preferred alternative, that is, the investment that has *preference* over others.

This chapter looks at the background to viability and preference calculations.

The appraisal of an investment is typically carried out in a systems analysis configuration (Carmichael, 2013). The cash flows are converted to measures of present worth, internal rate of return and so on, via compound interest (which is discussed later) based expressions. Broadly, the appraisal calculations might be referred to as *discounted cash flow (DCF) analysis*, when dealing with either cash flows, or benefits and costs in money units.

For the deterministic Part I, the terms *costs* and *benefits* will be used because they simplify explanations. Benefits and costs here are deterministic. However, for the probabilistic Part II, it is necessary to speak more generally in terms of cash flows. Cash flows then become random variables. It is possible to describe the deterministic case in terms of cash flows; however, this overly complicates the deterministic case.

In any investment, there will be costs. These are inputs to the investment. As a result of the investment, benefits then result (which may be positive benefits or negative disbenefits). These are outputs or outcomes of the investment. Both *viability* and *preference* can be best thought of in terms of investment inputs (costs) and investment outputs (benefits), that is, what the investor gets out of an investment with respect to what the investor puts into the investment.

2.1.1 Notation

The main notation adopted for the deterministic Part I is as follows:

i	time or period counter, $i = 0, 1, \ldots, n$; time may be measured in any unit, for example a day, a month or a year
n	lifespan; number of interest periods
r	interest rate (or discount rate) (expressed as a decimal, for example a rate of 5% per period is expressed as 0.05)
P	principal or present value
S_n	future value; the equivalent future amount of P accruing at a rate r for n periods
A	a uniform series amount
B	benefit
C	cost
PW	present worth
AW	annual worth
FW	future worth
IRR	internal rate of return
PBP	payback period
BCR	benefit:cost ratio

2.1.2 Viability

Commonly, a potential investment is said to be viable if the outcomes of an investment exceed what is put into the investment:

$$\text{Benefits} > \text{Costs}$$

Or expressed differently,

$$\text{Benefits} - \text{Costs} > 0$$

or

$$\frac{\text{Benefits}}{\text{Costs}} > 1$$

Additional measures of viability can be given. Later these additional measures are shown to be in terms of payback periods and interest rates. (In Part II, the definition of viability gets enlarged, when the benefits and costs become random variables.)

Viability here is a constraint (Carmichael, 2013), expressed in terms of benefits needing to exceed the costs. Satisfying the constraint means that the investment is viable; not satisfying the constraint means that the investment is nonviable.

2.1.3 Preference

Where multiple potential investments exist, a preferred investment is sought. Commonly, the preference is given to the one that maximizes the outcomes of the investment compared to what is put into the investment:

$$\text{Maximum (Benefits – Costs)}$$

or

$$\text{Maximum} \quad \frac{\text{Benefits}}{\text{Costs}}$$

Additional measures of preference can be given. Later these additional measures are shown to be in terms of payback periods and interest rates.

The difference (benefits minus costs), or the ratio (benefits:costs) are objective functions (Carmichael, 2013). The preferred investment maximizes the objective function.

2.1.4 Benefits and costs

The appraisal of an investment looks at the benefits and costs of everything related to the investment, both now and into the future.

Benefits and costs are looked at from the viewpoint of the relevant stakeholder or investor. Each stakeholder has a different set of benefits and costs. What may be a cost (or benefit) to one stakeholder may not be a cost (or benefit) to another. What may be a positive benefit to one stakeholder may be a negative benefit to another.

The distinction between benefits and costs is best made by regarding costs as input to the investment, while benefits (both positive benefits and negative disbenefits) are output or outcomes from the investment (Figure 2.1). That is, anything input to an investment is a cost, while anything resulting from the investment is a benefit (positive and negative).

Each stakeholder will have its own set of investment inputs and outputs. That is, a different appraisal applies for every different stakeholder.

Figure 2.1 Distinction between costs and benefits. (From Carmichael, D. G., *Problem Solving for Engineers*, Taylor & Francis/CRC Press, London, 2013.)

Typical costs (investment inputs) include the following:

- Initial invested capital; creation cost
- Design and construction costs
- Ongoing operation costs including maintenance, taxes, and insurance; money needed to run and sustain the investment
- Refurbishment, retrofitting or renovation costs during the lifespan of the investment
- Outlays, payments
- Disposal cost at the end of the life of the investment

Typical benefits (investment outputs) include both tangibles and intangibles:

- Things such as travel time or number of accidents, for a road
- Tolls and rental collected
- Salvage value or residual value, upon reaching the end of the life of the investment
- Product or income; anything arising from the investment
- Pollution, noise, loss of amenity, social disruption, loss of flora and fauna and similar (disbenefits or negative benefits)

The benefits may be gains (+) or losses (–) to the stakeholder. Negative benefits are referred to as disbenefits.

Note that a negative benefit is not the same as a cost – refer to Figure 2.1. Many texts and people get this wrong. In these texts, typically a benefit is defined as any 'gain', while a cost is defined as a 'loss'. And they get confused between costs and negative benefits, such that if something is not a 'gain', then it is classified (wrongly) as a cost. Be aware of this when reading the literature. Always refer back to Figure 2.1 for clarification as to what is a benefit and what is a cost.

2.1.5 Units of measurement

Commonly, benefits and costs are expressed in the same unit of measurement. Typically, this will be a money unit or money per something (for example money per kilolitre capacity for a water reservoir, or money per hectare of land improved for an agricultural investment), but it does not have to be (especially in the case of public sector investments). For example for agricultural investments, consumed water may be the unit of measurement. (On the other hand, appraisals for the private sector would almost certainly have a money and cash flow bias.)

Typically, benefits and costs are also translated to a common base, such as annual values (that is, per annum or abbreviated as p.a.) or present-day values, taking into account the time value of money.

Where intangibles need to be expressed in money units, shadow prices are developed; for example attempts are made at putting a monetary value on noise, aesthetics, human life, social disruption and parkland in order to bring them into the mathematical calculations. This usually involves value judgements and hence can be quite controversial. For example what is the monetary value of a human life, parkland or endangered animal?

Where the different intangibles remain expressed in their original nonmoney units or it is considered improper to express them in money units, they may take on the form of constraints (requirements that need to be satisfied), rather than entering the calculations as benefits and costs. Alternatively, these intangibles need to be traded off against the monetary side of the investment (Carmichael, 2013).

2.1.6 Time value of money

The life of investments can be long – 25, 50, 100 years or more – and a big influence on appraisal calculations comes from the fact that money has a time value. A dollar now is not the same as a dollar in the future, because money can earn interest over time.

Appraisals accordingly may take two forms:

- Appraisals that ignore the time value of money and use raw benefits and costs might be referred to as unadjusted or *nondiscounted*. They are performed using original values of benefits and costs, which occur at different times in the life of the investment.
- In appraisals that include the time value of money, the benefits and costs, which occur at different times in the life of the investment, for comparability, are translated (this is referred to as *discounted*) to the same or common time base, which is typically, but need not be, present-day money units or annual money units. The discounting is dependent on an *interest rate*, which *reflects the time value of money*.

A set of discounting equations is available for this second form. The associated mathematical analysis is referred to as *discounted cash flow (DCF) analysis*. The equations include reference to

- The duration or life of the investment
- An interest rate (Later, a related term – discount rate – is introduced.)
- Costs and benefits, and when they occur in time

Estimates for lifespan, for interest rate and for costs and benefits, in practice, may only be approximate, or may be estimated in a somewhat arbitrary manner, which makes any appraisal much more indefinite in reality than the mathematics would have people believe.

2.1.7 Cash flow

Cash flow refers to money coming in (*cash inflow*) and money going out (*cash outflow*). The *net cash flow* refers to the difference between cash inflow and cash outflow. The term *cash flow* has many uses, but in this book, it refers to the usage given here. Cash flow might be represented schematically as in Figure 2.2.

The cash outflows are costs and negative benefits (disbenefits), while the cash inflows are benefits.

An example cash flow diagram associated with a piece of infrastructure is shown in Figure 2.3.

2.1.8 Terminology uses

Be aware that terminology may be used in different ways by different people and different texts on the topic. Some examples are given here.

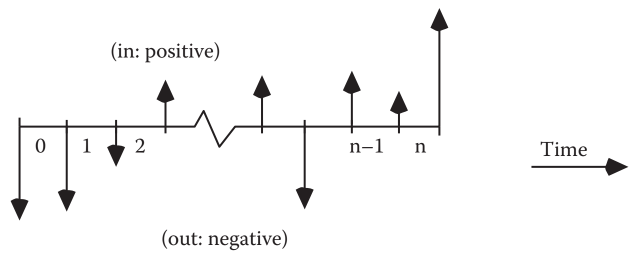

Figure 2.2 Cash flow diagram example.

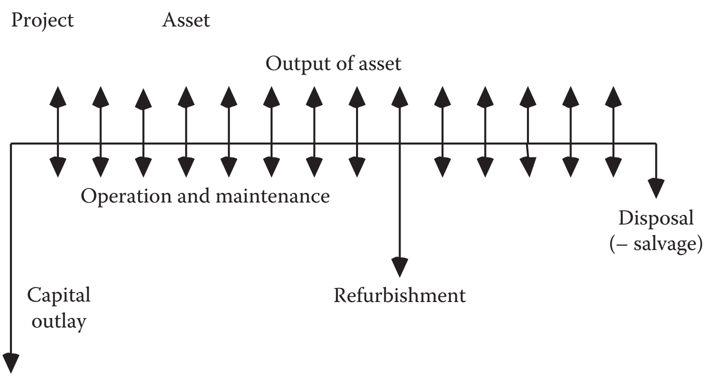

Figure 2.3 Example cash flow diagram for a piece of infrastructure; time runs from left to right.

1. Consider a project as an investment. Some people may use the term *project* to refer to all that is involved in bringing some asset into being. Others use the term to include the whole asset life cycle, or limit it to just the physical asset by itself. Many people use the term *project viability* when they mean 'project work that looks at the viability of the project and end-product of the project'. *Viability of project work* is an additional consideration, but one given less attention than *end-product viability*. In principle, the approach to looking at the viability of project work or viability of the end-product is the same; time horizons considered will be longer in the latter case, and of course the particular costs and benefits will be different.

   A project will commonly come about because of an identified need or want for some product, facility, asset, etc. This end-product is achieved through a project. However, as mentioned, the term project may also incorporate the full asset life cycle, or be the asset itself. For example a building might be a project to some people, while others might only consider a project up until the physical thing called a building comes into existence. The definition of a project is sufficiently flexible to include the operation and maintenance phase of a product within what is called the project (Carmichael, 2004).
2. The term *cash flow* may be used by some people to be equivalent to cash inflow only or cash outflow only, or the difference between cash inflow and cash outflow. Also, for example some people use this term to represent cumulative cash outflow or cumulative cash inflow. Clarification is needed as to what is intended when the term cash flow occurs in some texts and usage by people.

Confusion between these multiple usages of terminology does not appear to be a major concern to practitioners. Everything seems to work out in the end. However, be aware of multiple uses of technical terms. It would be nice to have one meaning for each word in the field of management, in order that management could progress, but this is not the reality.

2.1.9 Appraisal in stages

For large infrastructure or asset investments, appraisal might be undertaken in two or more stages. Basically, the same issues are considered in each appraisal stage, the degree of detail being least in the first stage and, as a result, involves less expenditure to undertake. Should the outcome of the first appraisal appear favourable, then the extra expense involved in the preparation of a more detailed appraisal can be justified.

There are a wide and varying number of activities, which have to be considered in a comprehensive appraisal. The order in which these activities are undertaken and their magnitude vary from investment to investment.

2.2 INTEREST

Interest reflects the time value of money; a dollar today is worth more than a dollar in the future. When money is borrowed, interest is payable to the lender. This interest may typically be calculated as a percentage of the money borrowed, and the percentage per period (usually per annum) is referred to as an *interest rate*. The interest rate represents the way money is worth more now than in the future. It allows future amounts to be converted (reduced, discounted) to present-day values.

Simple interest refers to a one-off interest payment, based on an invested amount of money (the principal). That is, at the end of the investment period, the invested amount of money gets repaid together with an interest payment based solely on the invested amount.

Compound interest refers to paying interest on interest owed, based on an invested amount of money (the principal). That is, at the end of the investment period, the invested amount of money gets repaid together with an interest payment based not only on the invested amount, but also the intermediate interest accrued.

Compounding (accruing) refers to an amount of money (the principal), subject to a given interest rate, accumulating periodically over time. That is, the invested amount grows because of interest accrued on the invested amount together with interest on the accrued interest.

Discounting refers to the relationship between the future value of an amount of money and its present value, based on assumptions about a periodic (usually annual) rate of interest and the number of compounding or interest periods (usually years). All future amounts of money can be converted to an equivalent present-day value.

In an appraisal, it is generally assumed that had money not been invested, then this money would have earned interest by being invested elsewhere. That is, if an investor uses its own money (sometimes referred to as *equity*), then interest should also be calculated on this, assuming that the money could have been invested elsewhere had it not been used in this investment. Of course, money borrowed from others (referred to as *debt*), will require interest payments at whatever interest rate is stated in the loan contract.

The following is extracted from *How to Lie with Statistics* (Huff, 1954, p. 136):

> Change-of-subject makes it difficult to compare cost when you contemplate borrowing money either directly or in the form of installment buying. Six per cent sounds like six per cent – but it may not be at all.
>
> If you borrow $100 from a bank at six per cent interest and pay it back in equal monthly installments for a year, the price you pay for

the use of the money is about $3. But another six per cent loan, on the basis sometimes called $6 on the $100, will cost you twice as much. That's the way most automobile loans are figured. It is very tricky.

The point is that you don't have the $100 for a year. By the end of six months you have paid back half of it. If you are charged $6 on the $100, or six per cent of the amount, you really pay interest at nearly twelve%.

Even worse was what happened to some careless purchasers of freezer-food plans in 1952 and 1953. They were quoted a figure of anywhere from six to twelve per cent. It sounded like interest, but it was not. It was an on-the-dollar figure and, worst of all, the time was often six months rather than a year. Now $12 on the $100 for money to be paid back regularly over half a year works out to something like forty-eight per cent real interest. It is no wonder that so many customers defaulted and so many food plans blew up.

2.2.1 Discount rate

The term *discount rate* may be preferred to be used by some people rather than *interest rate*, but mathematically they are treated the same in deterministic appraisal calculations. In the equations that follow, the symbol 'r' applies equally to interest rate and discount rate. Both interest rate and discount rate reflect the time value of money.

'Discount rate' is used to represent real change in value to the investor as determined by the possibilities for productive use of the money, and any risk associated with the use of that money. This partly implies that people should get more value from the money they borrow than the interest they pay on that borrowed money (or could receive on their own money); otherwise, there may be no point in borrowing (using) the money; that is, the discount rate chosen will be greater than the rate at which money is borrowed, or could be obtained on own funds if invested elsewhere.

Discount rates can vary from investment to investment, company to company, individual to individual. The choice of a discount rate is often a cause of disagreements in financial appraisal. One reason for this is a lack of universal agreement on how to establish its value. A second reason for this is that a discount rate is not a rational way to deal with uncertainty and risk; it is convenient to use and simplifies calculations by converting a probabilistic situation to a deterministic one, but it is nonetheless not rational.

The terms *cost of capital* and *cost of finance* and similar might be variously used by people to mean the rate applying to borrowed money. A weighted cost of capital might be used, where different weights are attached to money coming from different sources.

2.2.2 Discounting equations

In the following development, the equations and discussion make reference to an interest rate, but apply equally well to a discount rate. The discounting equations are first given for the simplest case (Section 2.2.3), namely the relationship between the value of a single amount of money now and its value in the future. This is then extended (Section 2.2.4) to give the relationship between the value of a single amount of money now and its future value spread out over the life of the investment. It is seen that the basis of all discounting equations is compound interest, manipulated to get equations into usable forms.

In a textbook treatment, particularly with textbook exercises, cash flows are usually prescribed to occur in given time periods. The question that then sometimes arises is, What happens if the cash flow occurs at the very start or very end of a period? This is only a concern in textbooks. In practice, the actual timing of any cash flow would be known, and it is that time which would be used in any calculations.

2.2.3 Single amount

Some equations that can be used to do most calculations in appraisal follow. Commonly, the time period for calculations is one year or one month, but any time period can be used. The notation of Section 2.1.1 is used.

Simple interest. An amount of money P, invested at a rate r, at the end of one period becomes,

$$S_1 = P(1 + r)$$

With simple interest calculations, interest is applied on the principal only. P might be referred to as the *principal*, but the equation is a general relationship between a future and present amount of money.

Compound interest. With compound interest calculations, interest is applied on the initial amount invested, as well as on the interest of previous periods.

An amount P accumulating at a rate r, at the end of n periods becomes,

$$S_n = P(1 + r)^n \tag{2.1}$$

See Figure 2.4. P, again, might be referred to as the *principal*, but the equation is a general relationship between a future and present amount of money. For n = 1, the earlier simple interest equation results.

$(1 + r)^n$ is termed the *compound amount factor* (caf).

Equation (2.1) may be rewritten as

$$P = \frac{S_n}{(1 + r)^n} \tag{2.2}$$

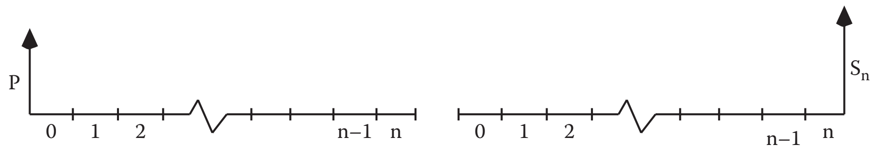

Figure 2.4 Equivalence – present value, P, and future value, S_n.

This gives the present value equivalent P to a future amount S_n occurring at period n.

$1/(1 + r)^n$ is termed the *present worth factor* (pwf).

Example

The present value of $1000 (occurring in year 5), at an interest rate of 10% per annum, assuming compound interest applies, is,

$$P = \frac{1000}{(1 + 0.1)^5} = \$620.92$$

2.2.4 Uniform series of amounts

For a uniform series amount, A, from Equation (2.2),

$$P = \frac{A}{(1 + r)} + \frac{A}{(1 + r)^2} + \ldots + \frac{A}{(1 + r)^n}$$

Multiplying both sides by (1 + r) gives,

$$P(1 + r) = A + \frac{A}{(1 + r)} + \ldots + \frac{A}{(1 + r)^{n-1}}$$

Subtracting the last two expressions and rearranging gives,

$$P = A \; \frac{(1 + r)^n - 1}{r(1 + r)^n} \qquad (2.3)$$

See Figure 2.5. This gives the present value of a uniform future series of amounts. Note, as $n \to \infty$, $P \to A/r$.

The term inside the square brackets is referred to as the *(series) present worth factor* (pwf) or present value factor.

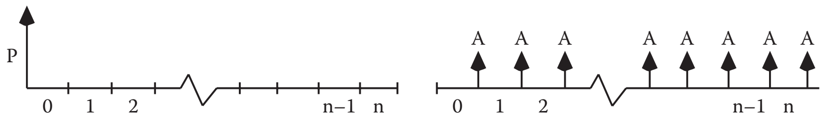

Figure 2.5 Equivalence – present value, P, and future uniform series amount, A.

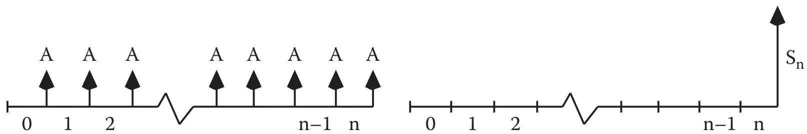

Figure 2.6 Equivalence – uniform series amount, A, and future value, S_n.

Rewriting Equation (2.3) gives,

$$A = P \; \frac{r(1+r)^n}{(1+r)^n - 1} \tag{2.4}$$

This gives the uniform future series equivalent of a present-day amount. Note, as $n \to \infty$, $A \to Pr$.

The term inside the square brackets is referred to as the *capital recovery factor* (crf).

Combining Equations (2.2) and (2.3) gives a relationship between S_n and A,

$$S_n = A \; \frac{(1+r)^n - 1}{r} \tag{2.5}$$

See Figure 2.6. This gives the future value of a uniform future series of amounts.

The term inside the square brackets is referred to as the *compound amount factor* (caf).

Rearranging Equation (2.5) gives,

$$A = S_n \; \frac{r}{(1+r)^n - 1} \tag{2.6}$$

This gives the uniform future series equivalent of a single future amount. The term inside the square brackets is referred to as the *sinking fund factor* (sff).

The above six discounting equations (Equations 2.1 to 2.6) are commonly used in appraisal. An appraisal usually involves capital expenditure, operating costs, income and so on (generally, benefits and costs), occurring at different times over the life of an investment. To obtain present values of these amounts of money, for the purpose of comparing to a common base (namely, time now, or present day), it is necessary to discount these future amounts of money using these equations. Certain additional assumptions might be made, namely that the same interest rate applies to all amounts, or different interest rates apply to cash inflows and cash outflows, and the interest rate(s) selected remains constant over the appraisal period.

2.2.5 Financial equivalence

The time value of money causes a certain present amount of money to be equal to different future amounts at different times in the future, and these change depending on the interest rate. And so it is possible to construct multiple financially equivalent scenarios, where the same present amount of money is equivalent to different future amounts, occurring at different times in the future.

2.2.6 Sensitivity of equations

The discounting and compounding equations (Equations 2.1 to 2.6) are sensitive to the interest rate, r, and the number of years, n, over which the equations are applied (as well as to any estimates made for P, A or S_n).

The present values of future amounts of money quickly become very small at nonnegligible interest rates. This means that any long-term future amounts (costs or benefits, but more usually this applies to benefits) can usually be neglected in an appraisal.

The present value of future amounts of money is sensitive to the interest rate. This sensitivity can influence preference for and against particular types of investments, depending on the future amounts involved and when in time they occur.

The present value of future amounts is sensitive to the period of appraisal selected, but more sensitive to the interest rate selected.

2.2.7 Special cases

Costs and benefits need not be characterized by a single amount or a series of uniform amounts, but might be characterized differently, for example as a uniform gradient series or geometric gradient series. It is almost certain that someone, somewhere has worked out discounting equations applying to each sort of different characterization. The so-called simulators that banks and building societies advertise to home loan customers would be particular examples. Many books have been published on the general topic. However for engineering/technical purposes, everything can be done with the above single and uniform series equations (Equations 2.1 to 2.6). The calculations might take a bit longer than using a special equation covering some different characterization, but by using a spreadsheet the calculations are relatively painless.

Exercises

2.1 How do you estimate the monetary value of a human life? What is the benefit of undertaking something that saves one human life per year? Do you calculate an average income over an average lifetime? Do you estimate average future contribution to a country's GDP? Do you calculate the average investment that the country has put into a person? Do you use insurance industry figures? Is a rich person worth more than a poor person? Is a lecturer worth more than a student? Is an educated person worth more than an uneducated one? Does your mother think that you are priceless, but everyone else thinks that you are worthless? Is there any rationality in any approach adopted?

2.2 The Rule of 72 is an old approximate method for estimating the time it will take to double an investment. Such a rule is useful because it allows for quick calculations, while being reasonably accurate. The number 72 is divisible by 2, 3, 4, 6, 8, 9 and 12 (and others), making it easy to work with manually. To calculate the time needed to double an investment, divide 72 by the interest rate. For example by investing $1 at 8% per annum, it will take 72/8 = 9 years for the investment to be worth $2. Alternative versions of this rule use 70 or 69 instead of 72. The rule may be extended to tripling (use 114) and quadrupling (use 144). With inflation, to determine the time for the buying power of money to halve, divide 72 by the inflation rate. For example at 3% per annum inflation, it will take 72/3 = 24 years for the value of a currency to halve. If the salary of an employee increases at a rate of 4% per annum, the salary will have doubled in 18 years (actual 17.67 years).

Examine the accuracy of the Rule of 72 for the following:

a. Interest rates per annum of 5%, 10%, 15%, 20%, 25%.
b. Using 70 or 69 instead of 72.

2.3 Plot the values obtained from the Rules of 72, 69, 70 and 76 against interest rate. Also plot the actual values. What do you conclude about these rules at different levels of interest rates up to say 20% per annum?